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Vol. 193 MAY, 1922 No. 5

Journal of The Franklin Institute

1826 1922

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Published Monthly by
THE FRANKLIN INSTITUTE OF THE STATE OF PENNSYLVANIA, PHILADELPHIA

SIX DOLLARS PER YEAR (Foreign Postage Additional.) SINGLE NUMBERS, SIXTY CENTS
Entered at the Post Office at Philadelphia, Pa., as second-class matter.
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Acceptance for mailing at special rate of postage provided for in section 1103, Act of October 3, 1917, authorized on July 3, 1919.

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Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 193 MAY, 1922 No. 5

ATOMIC WEIGHTS AND ISOTOPES.*

BY

F. W. ASTON, M.A., D.Sc., F.R.S.

Fellow of Trinity College Cambridge.

THAT matter is discontinuous and consists of discrete particles is now an accepted fact, but it is by no means obvious to the senses. The surfaces of clean liquids, even under the most powerful miscroscope, appear perfectly smooth, coherent and continuous. The merest trace of a soluble dye will color millions of times its volume of water. It is not surprising therefore that in the past there have arisen schools who believed that matter was quite continuous and infinitely divisible.

The upholders of this view said that if you took a piece of material, lead, for instance, and went on cutting it into smaller and smaller fragments with a sufficiently sharp knife, you could go on indefinitely. The opposing school argued that at some stage in the operations either the act of section would become impossible, or the result would be lead no longer. Bacon, Descartes, Gassendi, Boyle and Hooke were all partial to the latter theory, and Newton in 1675 tried to explain Boyle's Law on the assumption that gases were made up of mutually repulsive particles.

The accuracy of modern knowledge is such that we can carry

* Summary of a series of lectures given before The Franklin Institute, March 6–10, 1922, see page 712.

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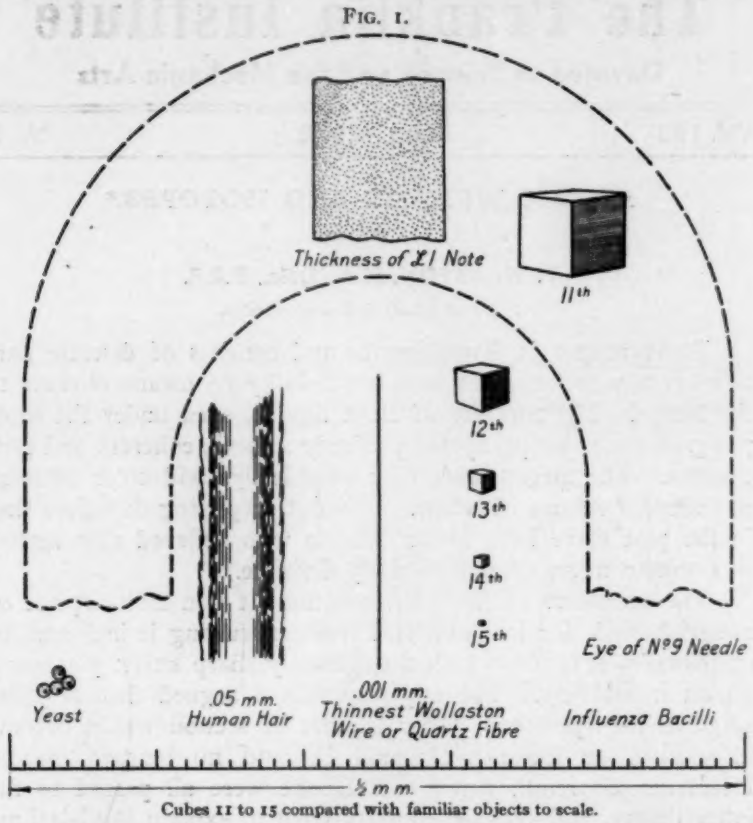
out, indirectly at least, the experiment suggested by the old philosophers right up to the stage when the second school is proved correct, and the ultimate atom of lead reached. For convenience, we will start with a standard decimetre cube of lead weighing 11.37 kilograms, and the operation of section will con-

FIG. 1.



Cubes 11 to 15 compared with familiar objects to scale.

sist of three cuts at right angles to each other, dividing the original cube into eight similar bodies each of half the linear dimensions and one-eighth the weight. Thus the first cube will have 5 cm. sides and weigh 1.42 kilograms, the second will weigh 178 grs., the fourth 2.78 grs. and so on. Diminution in the series is very rapid and the result of the ninth operation is a quantity of lead just weighable on the ordinary chemical balance. The results of further operations are compared with suitable objects

and a scale of length in Figs. 1, 2 and 3. The last operation possible, without breaking up the lead atom, is the twenty-eighth. The twenty-sixth cube is illustrated in Fig. 3. It contains 64 atoms, whose size, distance apart and general arrangement can be represented with considerable accuracy, thanks to the exact knowledge derived from research on X-rays and specific heats. On the same scale are represented the largest atom, cæsium, and the smallest atom, carbon, together with molecules of oxygen and nitrogen, at their average distance apart in the air, and the helical arrangement of silicon and oxygen atoms in quartz crystals discovered by X-ray analysis. The following table shows at what stages certain analytical methods break down. The great superiority of the microscope is a noteworthy point.

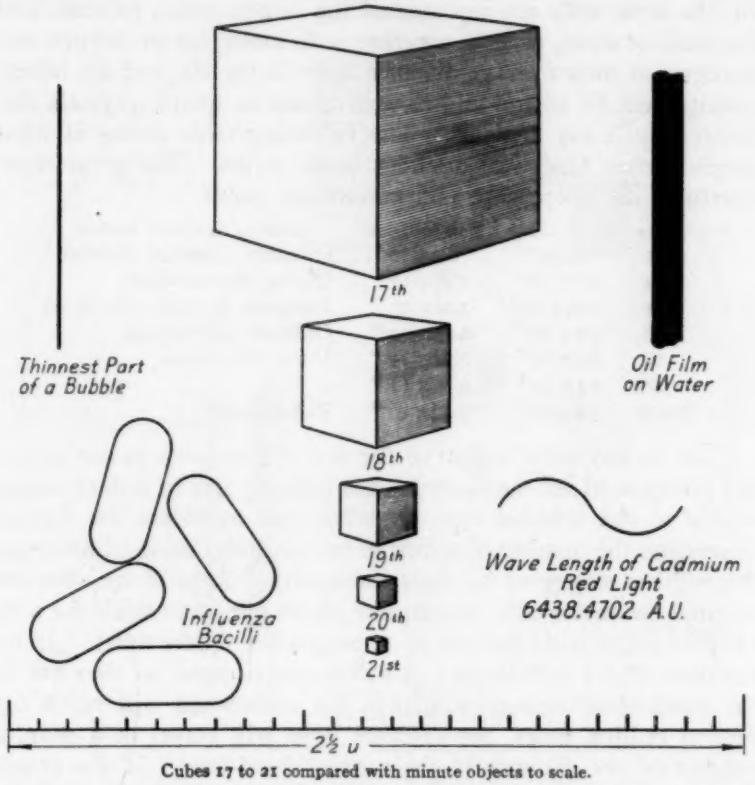
Cube.	Side in Cms.	Mass in Grms.	Limiting Analytical Method.
9	.0195	8.5×10^{-5}	Ordinary Chemical Balance
14	6.1×10^{-6}	2.58×10^{-9}	Quartz Micro-balance
15	3.05×10^{-6}	3.22×10^{-10}	Spectrum Analysis (Na lines)
18	3.8×10^{-7}	6.25×10^{-12}	Ordinary Microscope
24	6.0×10^{-8}	2.38×10^{-15}	Ultra Microscope
28	3.7×10^{-9}	5.15×10^{-20}	
Atom.	3.0×10^{-8}	3.44×10^{-22}	Radioactivity

Just as any vivid notion of the size of the cubes passes out of our power at about the twelfth—the limiting size of a dark object visible to the unaided eye—so when one considers the figures expressing the number of atoms in any ordinary mass of material, the mind is staggered by their immensity. Thus if we slice the original decimetre cube into square plates one atom thick the area of these plates will total one and one-quarter square miles. If we cut these plates into strings of atoms spaced apart as they are in the solid, these decimetre strings put end-to-end will reach 6.3 million million miles, the distance light will travel in a year, a quarter of the distance to the nearest fixed star. If the atoms are spaced but one millimetre apart the string will be three and a half million times longer yet, spanning the whole universe.

Again, if an ordinary evacuated electric light bulb were pierced with an aperture such that one million molecules of the air entered per second, the pressure in the bulb would not rise to that of the air outside for a hundred million years. Perhaps the most striking illustration is as follows: Take a tumbler of water and—supposing it possible—label all the molecules in it. Throw the water into the sea, or, indeed, anywhere you please, and after a

period of time so great that all the water on the earth—in seas, lakes, rivers and clouds—has had time to become *perfectly mixed,* fill your tumbler again at the nearest tap. How many of the labelled molecules are to be expected in it? The answer is, roughly, 2000; for although the number of tumblers full of

FIG. 2.



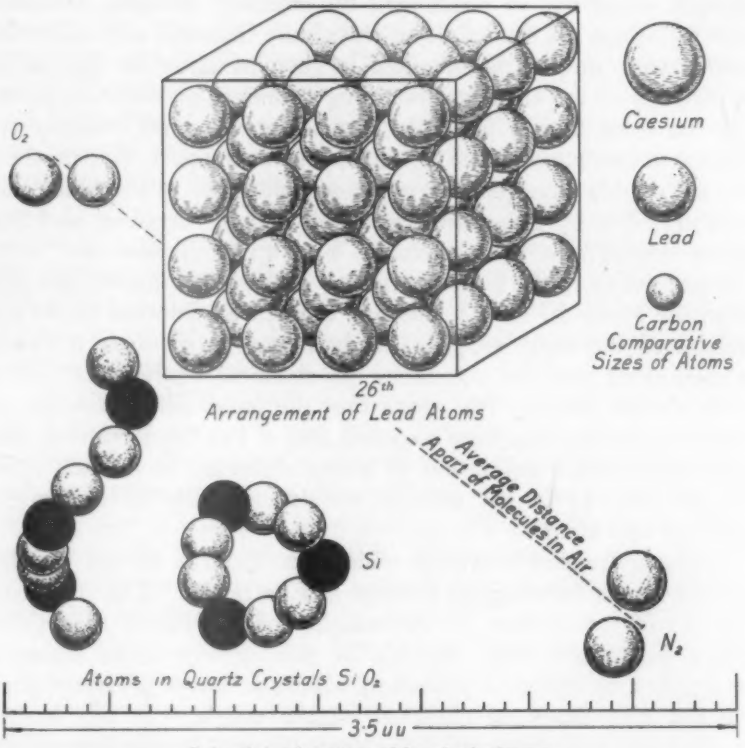
Cubes 17 to 21 compared with minute objects to scale.

water on the earth is 5×10^{21}, the number of molecules of water in a single tumbler is 10^{25}.

From the above statements it would, at first sight, appear absurd to hope to obtain effects from single atoms, yet this can now be done in several ways, and indeed it is largely due to the results of such experiments that the figures can be stated with so much confidence. Detection of an individual is only feasible in the case of an atom moving with an enormous velocity when, although its mass is so minute, its energy is quite appreciable.

The charged helium atom shot out by radioactive substances in the form of an alpha ray possesses so much energy that the splash of light caused by its impact against a fluorescent screen can be visibly detected, the ionization caused by its passage through a suitable gas can be measured on a sensitive electrometer and, in the beautiful experiments of C. T. R. Wilson, its path in air

FIG. 3.



Cube 26 showing atoms with scale of reference.

can be both seen and photographed by means of the condensation of water drops upon the atomic wreckage it leaves behind it.

In the first complete Atomic Theory put forward by Dalton in 1803 one of the postulates states that: "Atoms of the same element are similar to one another and equal in weight." Of course, if we take this as a definition of the word "Element" it becomes a truism, but, on the other hand, what Dalton probably meant by an element, and what we understand by the word to-day,

is a substance such as hydrogen, oxygen, chlorine, or lead, which has unique chemical properties and cannot be resolved into more elementary constituents by any known chemical process. For many of the well-known elements Dalton's postulate still appears to be strictly true, but for others, probably the majority, it needs some modification.

The idea that atoms of the same element are all identical in weight could not be challenged by ordinary chemical methods, for the atoms are by definition chemically identical and numerical ratios were only to be obtained in such methods by the use of quantities of the element containing countless myriads of atoms. At the same time it is rather surprising, when we consider the complete absence of positive evidence in its support, that no theoretical doubts were publicly expressed until late in the nineteenth century, first by Schutzenberger and then by Crookes, and that these doubts have been regarded, even up to the last few years, as speculative in the highest degree. In order to dismiss the idea that the atoms of such a familiar element as chlorine might not all be of the same weight, one had only to mention diffusion experiments and the constancy of chemical equivalents. It is only within the last few years that the lamentable weakness of such arguments has been exposed and it has been realized that the experimental separation of atoms differing from each other by as much as 10 per cent. in weight, is really an excessively difficult operation.

There are two ways by which the identity of the weights of the atoms forming an element can be tested. The one is by the direct comparison of the weights of individual atoms, the other is by obtaining samples of the element from different sources or by different processes, which, although perfectly pure, do not give the same chemical atomic weight. It was by the second and less direct of these methods that it was first shown that substances could exist which, though chemically identical, had different atomic weights.

In 1906 Boltwood,[1] at Yale, discovered a new element in the radioactive group which he called ionium, and described as having chemical properties similar to those of thorium. So much was this the case that if, by accident, salts of these two elements were mixed, he found it impossible to separate them again by any of the chemical processes. This chemical identity was confirmed in

the most convincing manner by the later work of Marckwald, Keetman [2] and Welsbach,[3] although the two elements certainly had different radioactive properties and it was extremely probable that they had different atomic weights. More identities of a similar nature among the radioactive elements were discovered by Soddy,[4] Hahn,[5] and others, and the situation in 1910 will be found admirably summed up by Soddy in his report to the Chemical Society for that year.[6] In 1912, Russel and Rossi [7] showed that the spectra of ionium and thorium were indistinguishable and Rutherford's theory of the "Nucleus Atom," supplied a possible explanation. The association of the chemical and spectroscopic properties of an element with something more fundamental than its atomic weight, namely the charge on the nuclei of its atoms or its "Atomic Number," was proved by the epoch-making work of Moseley in 1913.[8] This idea gave a simple and entirely satisfactory meaning to the chemical laws of the radioactive disintegrations discovered a little earlier and predicted that among the numerous products of these disintegrations there must of necessity be some having identical chemical properties but different atomic weights.

To the latter the name "Isotopes" was applied by Soddy in the following words: "The same algebraic sum of the positive and negative charges in the nucleus when the arithmetical sum is different gives what I call 'isotopes' or 'isotopic elements' because they occupy the same place in the periodic table. They are chemically identical, and save only as regards the relatively few physical properties which depend upon atomic mass directly, physically identical also."

The theory of isotopes received its most triumphant vindication, as far as it concerned the products of radioactivity, from the results of work on the atomic weight of lead. Study of the radioactive disintegrations shows that the final product of every series is lead. If we take the main chain of the uranium-radium transformation this lead must have an atomic weight 206, for it has lost 5 alpha particles—each of weight 4—since it was radium, and the atomic weight of radium is 226. On the other hand, if we take the main thorium chain the lead end product must be 6 alpha particles lighter than thorium (232.15) and so should have an atomic weight about 208.

Now ordinary lead, from non-radioactive sources has an

atomic weight 207.20, so Soddy[9] suggested in 1913 that the lead derived from minerals containing uranium but no thorium might have a smaller atomic weight than ordinary lead, and on the other hand, the atomic weight of lead from minerals containing thorium but no uranium might be greater.

The first experiments were made by Soddy and Hyman[10] with a very small quantity of lead from Ceylon thorite. This gave a perceptibly higher atomic weight than ordinary lead. Later a large quantity of the same mineral was available. The lead from this when carefully purified gave a density 0.26 per cent. higher than that of common lead. On the assumption that the atomic volumes of isotopes are equal, this figure corresponds to an atomic weight of 207.74. A chemical atomic weight determination gave 207.694. A sample of the same lead was sent to Vienna where Professor Honigschmid, a well-known expert in such matters, obtained from it a value 207.77 as a mean of eight determinations. These figures not only showed that thorium lead had a higher atomic weight than ordinary lead, but also that their atomic volumes were identical, as expected from theory.[11]

	Common Lead. A	Mixture Australian. B	Uranio-Lead. C	Percentage Difference. A–B	A–C
Atomic weight	207.19	206.34	206.08	0.42	0.54
Density	11.337	11.280	11.273	0.42	0.56
Atomic volume	18.277	18.278	18.281	0.01	0.02
Melting point (absolute)	600.53	600.59	—	0.01	—
Solubility (of nitrate)	37.281	37.130	—	0.41	—
Refractive index (nitrate)	1.7815	1.7814	—	0.01	—
Thermoelectric effect	—	—	—	0.00	—
Spectrum wave-length	—	—	—	0.00	0.00

At the same time, as this work was in progress, the leading American authority on atomic weights, T. W. Richards,[12] of Harvard, started a series of investigations on lead derived from various radioactive minerals. The samples of lead from uranium minerals all gave results lower than ordinary lead, as was expected, and one particularly pure specimen of uranio-lead from Norwegian cleveite gave 206.08,[13] a very striking agreement with theory. The following table of properties is taken from his Presidential address to the American Association at Baltimore, December, 1918.

In further confirmation Maurice Curie [14] in Paris reported 206.36 for a lead from carnotite, and a still lower figure, 206.046, was obtained by Honigschmid in Vienna for a lead from the very pure crystallized pitchblende from Morogoro.[15] This is the lowest atomic weight found so far. The highest, 207.9, was also determined by Honigschmid for lead from Norwegian thorite.

In the absence of the special radioactive evidence which can be used in special cases such as that of lead, the presence of isotopes among the inactive elements can only be detected by the direct measurement of the masses of individual atoms. This can be done by the analysis of positive rays.

The condition for the development of these rays is briefly ionization at low pressure in a strong electric field. Ionization, which may be due to collisions or radiation, means in its simplest case the detachment of one electron from a neutral atom. The two resulting fragments carry charges of electricity of equal quantity but of opposite sign. The negatively charged one is the electron, the atomic unit of negative electricity itself, and is the same whatever the atom ionized. It is extremely light and therefore in the strong electric field rapidly attains a high velocity and becomes a cathode ray. The remaining fragment is clearly dependent on the nature of the atom ionized. It is immensely more massive than the electron, for the mass of the lightest atom, that of hydrogen, is about 1850 times that of the electron, and so will attain a much lower velocity under the action of the electric field. However, if the field is strong and the pressure so low that it does not collide with other atoms too frequently, it will ultimately attain a high speed in a direction opposite to that of the detached electron, and become a "positive ray." The simplest form of positive ray is therefore an atom of matter carrying a positive charge and endowed, as a result of falling through a high potential, with sufficient energy to make its presence detectable. Positive rays can be formed from molecules as well as atoms, so that it will at once be seen that any measurement of their mass will give us direct information as to the masses of atoms of elements and molecules of compounds, and that this information will refer to the atoms or molecules *individually*, not, as in chemistry, to the mean of an immense aggregate. It is on this account that the accurate analysis of positive rays is of such importance.

In order to investigate and analyze them it is necessary to obtain intense beams of the rays. This can be done in several ways. The one most generally available is by the use of the discharge in gases at low pressure.

It is somewhat remarkable that notwithstanding the immense amount of research work done on the discharge at low pressure its most obvious phenomena are almost entirely without explanation; modern measurements and other data have merely destroyed the older theories without, as yet, suggesting others to replace them. In discussing positive rays it is of importance to consider the phenomena taking place immediately in front of the cathode of the discharge tube.

The comparatively dimly lit space between the cathode and the bright "negative glow" is named after its discoverer the "Crookes' Dark Space." Its length is roughly inversely proportional to the pressure of the gas in the tube. Its boundary, the edge of the negative glow, is remarkably sharp in most gases, quite amazingly so in pure oxygen. If large plane cathodes are used so that the effect of the glass walls—so far a complete mystery—does not come in, very accurate and consistent measurements of the Crookes' dark space can be made. Working with a very large "guard-ring" cathode, the writer showed in 1907 [16] that its length D could be expressed as

$$D = \frac{A}{P} + \sqrt{\frac{B}{i}}$$

where P is the pressure, i the current density on the surface of the cathode and A and B constants. This expression is fairly exact for ordinary gases, but only approximate for those of the helium group.[17] A varies both with the nature of the gas and the metal used as cathode. With the same metal as cathode it is four to five times as big for hydrogen as for oxygen; using the same gas, it is about twice as big with a silver cathode as for one of aluminum.[18] The value of the remarkable constant B, which is independent of the pressure, is also practically unchanged either by the nature of the gas, or the nature or even the shape of the cathode.

During these investigations it was also demonstrated that in the absence of the positive column, the whole of the potential

V required to maintain the discharge, takes place between the cathode and the negative glow, and its relation to the current may be expressed by the equation

$$V = E + \frac{F\sqrt{i}}{P}$$

E and F being constants which depend on the nature of the gas and the metal used as cathode.

Measurements of the distribution of potential in the dark space by a method which appears free from objection [19] show that the field is practically zero in the negative glow itself and increases in a linear manner as we move towards the cathode. This means that the density of the positive electrification (excess of positive over negative ions) in the dark space is constant.

No theory yet put forward can account for these numerical relations; one can, however, be certain that ionization is going on at all points throughout the dark space, and that it reaches a very high intensity in the negative glow.[20] This ionization is probably caused for the most part by electrons liberated from the surface of the cathode (cathode rays). These, when they reach a speed sufficient to ionize by collision, liberate more free electrons which, in their turn, become ionizing agents, so that the intensity of ionization from this cause will tend to increase as we move away from the cathode.

The liberation of the original electrons from the surface of the cathode is generally regarded as due to the impact of the positive ions (positive rays) generated in the negative glow and the dark space. Even this idea, for which there is a fair amount of definite evidence, is now called in question for Ratner [21] has recently described experiments proving "that the initial discharge of electricity through vacuum tubes is not brought about by the impact of positive ions against the surface of the cathode, and that positive ions impinging upon the cathode with velocities corresponding to a fall through a potential difference up to 2000 volts, are unable to liberate electrons from the surface of the cathode." It must, however, be borne in mind that the nature of the positive ions used in Ratner's experiments is not known with certainty, and that the intensity of bombardment was of an entirely different and smaller order than that usually associated with normal discharge.

During the work on the length of the dark space a very curious and interesting phenomenon was observed in hydrogen and all the gases of the helium group. This consisted of a dark space very small and very dark, immediately in front of the cathode, inside the Crookes' dark space.[22] It can only be clearly seen by looking across the face of a large plane cathode when its appearance in pure helium or neon is very striking. Its properties are completely different to those of the Crookes' dark space. Its length, which is usually less than a millimetre, is independent of the pressure and varies roughly inversely as the square root of the current density. As the field close to the cathode varies directly as the square root of the current density this suggested at once that the new dark space represented a definite constant fall of potential. There is now little doubt that this "Primary Dark Space," as it has been called, defines the distance through which an electron, starting from rest at the surface of the cathode, must fall before it acquires sufficient energy to ionize the gas by collision. This explanation is supported in a striking manner by the appearance of the phenomenon when the current density is greatly reduced. Under these conditions alternate dark and bright bands, equally spaced, appear in front of the cathode exactly as would be expected from the production of successive generations of ions. Further support is given by the measurements in hydrogen and helium which indicate that the actual fall of potential across the primary dark space is twice as great for helium as for hydrogen and, on certain assumptions, is approximately equal to the ionization potential of these gases determined by other means.

In addition to cathode ray ionization the positive rays travelling towards the cathode themselves are capable of ionizing the gas, and radiation may also play an important part in the same process. The surface of the cathode will therefore be under a continuous hail of positively charged particles. Their masses may be expected to vary from that of the lightest atom to that of the heaviest molecule capable of existence in the discharge tube, and their energies from an indefinitely small value to a maximum expressed by the product of the charge they carry $\times$ the total potential applied to the electrodes. The latter is practically the same as the fall of potential across the dark space. If the cathode be pierced the rays pass through the aperture and form

a stream heterogeneous both in mass and velocity which can be subjected to examination and analysis.

In Sir J. J. Thomson's [23] "parabola" method of analysis of positive rays the particles after reaching the surface of the cathode enter a long and very fine metal tube. By this means a narrow beam of rays is produced which is subjected to deflection by electric and magnetic fields and finally falls upon a screen of fluorescent material or a photographic plate. The fields are arranged so that the two deflections are at right angles to each other. If we call the displacement on the plate due to the electric field x and that due to the magnetic field y for any particle (x, y) will be the rectangular coördinates of the point where it strikes the plate. Simple dynamics show that if the angle of deflection is small, for a particle of mass m, charge e and velocity v, the electric deflection $x = k(Xe/mv^2)$ and the magnetic deflection $y = k'(He/mv)$ where X and H are the magnetic and electric fields and k and k' constants depending solely on the dimensions and form of the apparatus. Hence if both fields are on together the locus of impact of all particles of the same $\frac{e}{m}$ but varying velocity will be a parabola. Since e must be the electronic charge, or a simple multiple of it, measurement of the relative positions of the parabolas on the plate enables us to calculate the relative masses of the particles producing them, that is, the masses of the individual atoms. The fact that the streaks were definite, sharp parabolas, and not mere blurs, constituted the first direct proof that atoms of the same element were, even approximately, of equal mass.

Many gases were examined by this method and some remarkable compounds, such as H_3, discovered by its means. When in 1912 neon was introduced into the discharge tube, it was observed to exhibit an interesting peculiarity. This was that whereas all elements previously examined gave single, or apparently single, parabolas, that given by neon was definitely double. The brighter curve corresponded roughly to an atomic weight of 20, the fainter companion to one of 22, the atomic weight of neon being 20.20. In consequence of reasoning adduced from the characteristics of the line 22, Sir J. J. Thomson was of the opinion that it could not be attributed to any compound, and that therefore it represented a hitherto unknown elementary constituent of neon.

This agreed very well with the idea of isotopes which had just been promulgated, so that it was of great importance to investigate the point as fully as possible.

The first line of attack was an attempt at separation by fractional distillation over charcoal cooled with liquid air, but even after many thousands of operations the result was entirely negative. The density of the fractions was determined by means of a special quartz microbalance [24] and was found to agree in every case with the accepted atomic weight of 20.20 [25] to an accuracy of 1 in 1000.

The second method employed was that of fractional diffusion through pipeclay, which after months of arduous work gave a small but definite positive indication of separation. A difference of about 0.7 per cent.[26] between the densities of the heaviest and lightest fractions was obtained. If V is the initial and v the residual volume of gas, Rayleigh's [27] theory of diffusion gives an enrichment of approximately $(V/v)^{1/21}$ in respect to the heavier constituent in the residue in the case of neon. This is the result of a single operation and the effective ratio V/v can be increased indefinitely by repeated operations, in these experiments it amounted to several thousand, but from the nature of the work cannot be stated exactly. If, to be perfectly safe, we take V/v as between 500 and 10,000, the theoretical increase in density of the heaviest fraction over normal should lie between 0.34 and 0.55 per cent. It was actually 0.4, quite a reasonable figure considering the inefficiency of the method. The decrease in density in the lightest fraction was rather less owing to the loss of a portion of the gas during the experiment. When the war interrupted the research, it might be said that several independent lines of reasoning pointed to the conclusion that neon was a mixture of isotopes, but none of these could be said to carry absolute conviction.

By the time the work was resumed in 1919 the existence of isotopes among the products of radioactivity had been put beyond all reasonable doubt by the work on the atomic weight of lead and was accepted generally. This fact automatically increased both the value of the evidence of the complex nature of neon and the urgency of its definite confirmation. It was realized that separation could only be very partial at the best and that the most satisfactory proof would be afforded by measurements of atomic weight by the method of positive rays. These would have to be

so accurate as to prove beyond dispute that the accepted atomic weight lay between the real atomic weights of the constituents, but corresponded with neither of them.

The parabola method of analysis was not sufficient for this, but the required accuracy was achieved by the arrangement illustrated diagrammatically in Fig. 4.[28] The exact mathematical analysis has now been worked out by R. H. Fowler,[29] but it is proposed to give only the approximate theory here for the sake of simplicity.

The rays after arriving at the cathode face pass through two very narrow parallel slits of special construction S_1 S_2, and the resulting thin ribbon is spread out into an electric spectrum by means of the parallel plates P_1, P_2. After emerging from the electric field the rays may be taken, to a first order of approximation, as radiating from a virtual source Z half-way through the field on the line S_1 S_2. A group of these rays is now selected by means of the diaphragm D, and allowed to pass between the parallel poles of a magnet. For simplicity the poles are taken as circular, the field between them uniform and of such sign as to bend the rays in the opposite direction to the foregoing electric field.

If θ and φ be the angles (taken algebraically) through which the selected beam of rays is bent by passing through fields of strength X and H, then

$$\theta v^2 = lX\frac{e}{m}\ (1), \text{ and } \varphi v = LH\frac{e}{m}\ (2),$$

where l, L are the lengths of the paths of the rays in the fields. Equation (1) is only true for small angles, but exact enough for practice. It follows that over the small range of θ selected by the diaphragm θv^2 and φv are constant for all rays of given e/m, therefore

$$\frac{\delta\theta}{\theta} + \frac{2\delta v}{v} = 0, \text{ and } \frac{\delta\varphi}{\varphi} + \frac{\delta v}{v} = 0,$$

so that

$$\frac{\delta\theta}{\theta} = \frac{2\delta\varphi}{\varphi},$$

when the velocity varies in a group of rays of given e/m. This equation appears correct within practical limits for large circular pole-pieces.

Referred to axes OX, OY the focus is at $r\cos(\varphi-2\theta')$, $r\sin(\varphi-2\theta')$, or $r, b.2\theta'$; so that to a first-order approximation, whatever the fields, so long as the position of the diaphragm is fixed, the foci will all lie on the straight line ZF drawn through Z parallel to OX. For purposes of construction G the image of Z in OY is a convenient reference point, φ being here equal to $4\theta'$. It is clear that a photographic plate, indicated by the thick line, will be in fair focus for values of e/m over a range large enough for accurate comparison of masses.

Since it is a close analogue of the ordinary spectrograph and gives a "spectrum" depending upon mass alone, the instrument

FIG. 4.

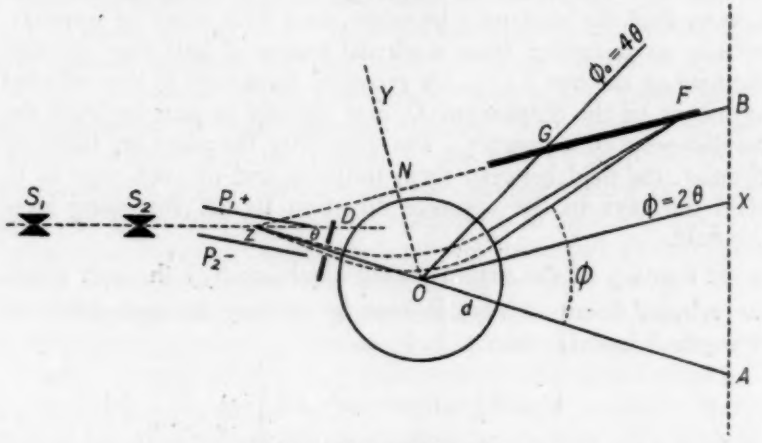
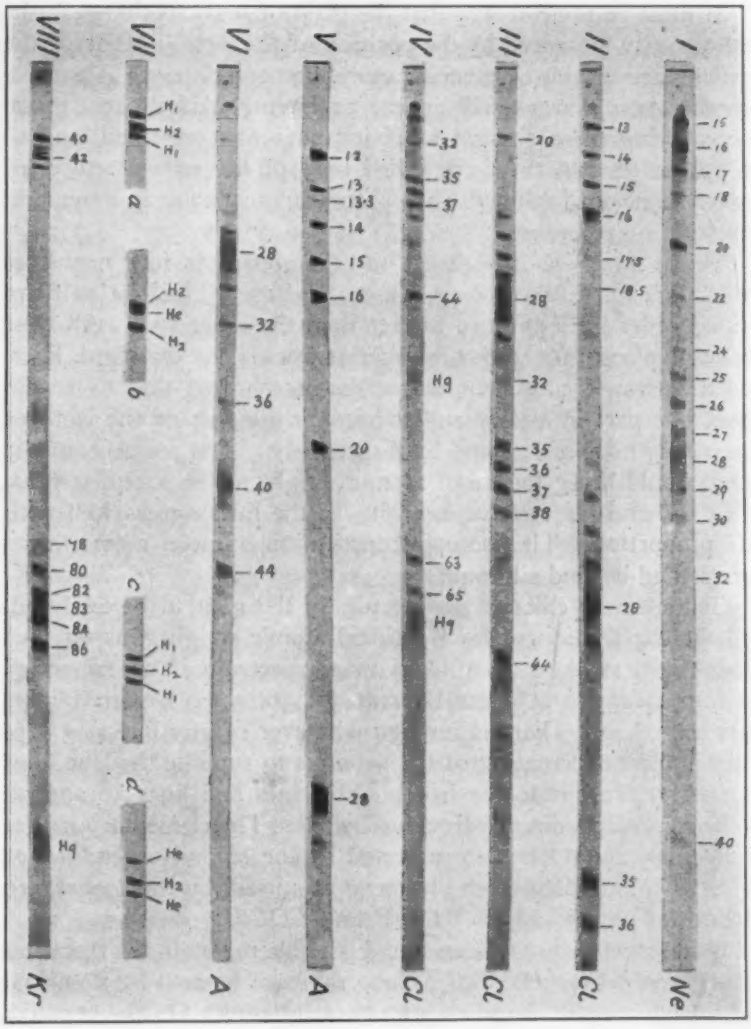


Diagram of mass-spectrograph.

is called a "mass-spectrograph" and the spectrum it produces a "mass-spectrum."

Fig. 5 shows a number of typical mass-spectra obtained by this means. The numbers above the lines indicate the masses they correspond to on the scale $O=16$. It will be noticed that the displacement to the right with increasing mass is roughly linear. The measurements of mass made are not absolute, but relative to lines which correspond to known masses. Such lines due to hydrogen, carbon, oxygen and their compounds are generally present as impurities or purposely added, for pure gases are not suitable for the smooth working of the discharge tube. The two principal groups of these reference lines are the C_1 group due to

FIG. 5.



Typical mass-spectra.

C (12), CH (13), CH_2 (14), CH_3 (15), CH_4 or O (16), and the C_2 group (24 to 30) containing the very strong line C_2H_4 or CO (28). These groups will be seen in several of the spectra

reproduced, and they give, with the CO_2 line (44), a very good scale of reference.

It must be remembered that the ratio of mass to charge is the real quantity measured by the position of the lines. Many of the particles are capable of carrying more than one charge. A particle carrying two charges will appear as having half its real mass, one carrying three charges as if its mass was one-third, and so on. Lines due to these are called lines of the second and third order. Lines of high order are particularly valuable in extending our scale of reference.

When neon was introduced into the apparatus four new lines made their appearance at 10, 11, 20 and 22. The first pair are second order lines and are fainter than the other two. All four are well placed for direct comparison with the standard lines, and a series of consistent measurements showed that to within about one part in a thousand the atomic weights of the isotopes composing neon are 20 and 22 respectively. Ten per cent. of the latter would bring the mean atomic weight to the accepted value of 20.20, and the relative intensity of the lines agrees well with this proportion. The isotopic constitution of neon seems therefore settled beyond all doubt.

The element chlorine was naturally the next to be analyzed, and the explanation of its fractional atomic weight was obvious from the first plate taken. Its mass spectrum is characterized by four strong first order lines at 35, 36, 37, 38, with fainter ones at 39, 40. There is no sign whatever of any line at 35.46. The simplest explanation of the group is to suppose that the lines 35 and 37 are due to the isotopic chlorines and lines 36 and 38 to their corresponding hydrochloric acids. The elementary nature of lines 35 and 37 is also indicated by the second order lines at 17.5, 18.5, and also, when phosgene was used, by the appearance of lines at 63, 65, due to $COCl^{35}$ and $COCl^{37}$.

Quite recently it has been found possible to obtain the spectrum of negatively charged rays. These rays are formed by a normal positively charged ray picking up two electrons. On the negative spectrum of chlorine only two lines, 35 and 37, can be seen, so that the lines at 36 and 38 cannot be due to isotopes of the element. These results, taken with many others which cannot be stated here in detail, show that chlorine is a complex element, and that its principal isotopes are of atomic weight 35 and 37. There may be,

in addition, a small proportion of a third of weight 39, but this is doubtful. Spectra II, III and IV show the results with chlorine taken with different magnetic field strengths.

The mass spectrum of argon shows an exceedingly bright line at 40, with second order line at 29, and third order line at $13^1/_3$. The last is particularly well placed between known reference lines, and its measurement showed that the triply-charged atom causing it had a mass 40 very exactly. Now the accepted atomic weight of argon is less than 40, so the presence of a lighter isotope was suggested. This was found at 36, and has now been fully substantiated; its presence to the extent of about 3 per cent. is sufficient to account for the mean atomic weight obtained by density determinations.

The elements hydrogen and helium present peculiar difficulties since their lines are so far removed from the ordinary reference scale, but, as the lines were expected to approximate to the terms of the geometrical progression 1, 2, 4, 8, etc., the higher terms of which are known, a special method was adopted by which a two to one relation could be tested with some exactness. Two sets of accumulators were selected, each giving very nearly the same potential of about 250 volts. The potentials were then made exactly equal by means of a subsidiary cell and a current-divider, the equality being tested to well within 1000 by means of a null instrument. If exposures are made with such potentials applied to the electric plates first in parallel and then in series, the magnetic field being kept constant, all masses having an exact two to one relation will be brought into coincidence on the plate. Such coincidences cannot be detected on the same spectrum photographically; but if we first add and then subtract a small potential from one of the large potentials, two lines will be obtained which closely bracket the third. To take an actual instance—using a gas containing hydrogen and helium, with a constant current in the magnet of 0.2 ampere, three exposures were made with electric fields of 250, $500+12$, and $500-12$ volts, respectively. The hydrogen molecule line was found symmetrically bracketed by a pair of atomic lines (Spectrum VII, *a* and *c*), showing within experimental error that the mass of the molecule is exactly double the mass of the atom. When after a suitable increase of the magnetic field the same procedure was applied to the helium line and that of the hydrogen molecule, the bracket was no longer

symmetrical (Spectrum VII, *b*), nor was it when the hydrogen molecule was bracketed by two helium lines (*d*). Both results show in an unmistakable manner that the mass of *He* is less than twice that of H_2. In the same way *He* was compared with O^{++} and H_3. The values obtained by its use can be checked in the ordinary way by comparing *He* with C^{++} and H_3 with *He*, these pairs being close enough together for the purpose. The following table gives the range of values obtained from the most reliable plates:

Line.	Method.	Mass assumed.	Mass deduced.
He	Bracket	$O^{++} = 8$	3.994–3.996
	Direct	$C^{++} = 6$	4.005–4.010
H_3	Bracket	$C^{++} = 6$	3.025–3.027
	Direct	He = 4	3.021–3.030
H_2	Bracket	He = 4	2.012–2.018

From these figures it is safe to conclude that hydrogen is a simple element and that its atomic weight, determined with such consistency and accuracy by chemical methods, is the true mass of its atom.[30]

The heavy inert gases give interesting and complicated results. Krypton is characterized by a remarkable group of five strong lines at 80, 82, 83, 84, 86, and a faint sixth at 78. This cluster of isotopes is beautifully reproduced with the same relative values of intensity in the second, and fainter still in the third order. These multiply-charged clusters give most reliable values of mass, as the second order can be compared with *A* (40) and the third with *CO* or N_2 (28) with the highest accuracy. It will be noted that one member of each group is obliterated by the reference line, but not the same one. The singly and doubly charged krypton clusters can be seen to the right and left of Spectrum VIII. It will be noticed that krypton is the first element examined which shows unmistakable isotopes differing by one unit only.

Xenon gives five strong lines 129, 131, 132, 134, 136 and some indication of two faint and doubtful ones 128 and 130.

Mercury probably consists of six isotopes. Its first, second, third, and higher order lines appear as a series of characteristic groups around positions corresponding to masses 200, 100,

$66^2/_3$, etc. Some of these will be easily distinguished on the spectra reproduced.

In addition to those mentioned above the elements boron, fluorine, silicon, bromine, sulphur, phosphorus, iodine and nickel have been analyzed by the mass-spectrograph, using the discharge tube method for producing their positive rays. Selenium, tellurium, antimony and tin have been attacked but not yet successfully.

Positive rays of the metallic elements cannot, in general, be obtained by the discharge tube method, but require special devices for their production. Thus the isotopic nature of lithium was first demonstrated by the use of anode rays derived from anodes containing salts of that metal.[31] Later by applying the mass-spectrograph to them all the other alkali metals were successfully analyzed.[32] Sodium and cæsium appear to be simple elements whilst lithium, potassium and rubidium each consist of two isotopes.

A powerful and ingenious method of generating positive rays of metallic elements has been worked out and used with great success by Dempster at Chicago.[33] He employs the element in the metallic state and ionizes its vapor by means of a subsidiary beam of cathode rays. The ions so produced are allowed to fall through a definite potential and being therefore of constant energy can be analyzed by the use of a magnetic field alone. By this arrangement Dempster discovered the three isotopes of magnesium [34] and confirmed those of lithium. A full account of this work has lately appeared.[35] Still more recently he has obtained results with calcium and zinc which indicate that the former consists almost entirely of an isotope 40 with probable traces of another 44, and that the latter has four isotopes—64, 66, 68 and 70. Since the vast majority of the elements not yet analyzed are metals Dempster's method is likely to yield enormously important results in the future. A complete list of the isotopes of the non-radioactive elements so far discovered is given on the following page.

By far the most important general result of these investigations is that, with the exception of hydrogen, the weights of the atoms of all the elements measured, and therefore almost certainly of all elements are whole numbers to the accuracy of

experiment. In the majority of the figures obtained by means of the mass-spectrograph this accuracy is one part in a thousand. Of course the error expressed in fractions of a unit increases

Table of Elements and Isotopes.

Element.	Atomic number.	Atomic weight.	Minimum number of isotopes.	Masses of isotopes in order of intensity.
H	1	1.008	1	1.008
He	2	4.00	1	4
Li	3	6.94	2	7, 6
Be	4	9.1	1	9
B	5	10.9	2	11, 10
C	6	12.00	1	12
N	7	14.01	1	14
O	8	16.00	1	16
F	9	19.00	1	19
Ne	10	20.20	2	20, 22, (21)
Na	11	23.00	1	23
Mg	12	24.32	3	24, 25, 26
Si	14	28.3	2	28, 29, (30)
P	15	31.04	1	31
S	16	32.06	1	32
Cl	17	35.46	2	35, 37, (39)
A	18	39.88	2	40, 36
K	19	39.10	2	39, 41
Ni	28	58.68	2	58, 60
As	33	74.96	1	75
Br	35	79.92	2	79, 81
Kr	36	82.92	6	84, 86, 82, 83, 80, 78
Rb	37	85.45	2	85, 87
I	53	126.92	1	127
X	54	130.2	5, (7)	129, 132, 131, 134 136, (128, 130?)
Cs	55	132.81	1	133
Hg	80	200.6	(6)	(197-200), 202 204
Dempster's latest results.				
Ca	20	40.07	(2)	40 (44)
Zn	30	65.37	(4)	(64, 66, 68, 70)

with the weight measured, but with the lighter elements the divergence from the whole number rule is extremely small.

This enables the most sweeping simplifications to be made in our ideas of mass. The original hypothesis of Prout, put forward in 1815, that all atoms were themselves built of atoms of protyle, a hypothetical element which he tried to identify with

hydrogen, is now reëstablished, with the modification that the primordial atoms are of two kinds: Protons and electrons, the atoms of positive and negative electricity.

The Rutherford atom, whether we take Bohr's or Langmuir's development of it, consists essentially of a positively charged central nucleus around which are set planetary electrons at distances great compared with the dimensions of the nucleus itself.

As has been stated, the chemical properties of an element depend solely on its atomic number, which is the charge on its nucleus expressed in terms of the unit charge, e. A neutral atom of an element of atomic number N has a nucleus consisting of $K+N$ protons and K electrons, and around this nucleus are set N electrons. The weight of an electron on the scale we are using is 0.0005, so that it may be neglected. The weight of this atom will therefore be $K+N$, so that if no restrictions are placed on the value of K any number of isotopes are possible.

A statistical study of the results given above shows that the natural restrictions can be stated in the form of rules as follows:

In the Nucleus of an Atom There is Never Less Than One Electron to Every Two Protons.—There is no known exception to this law. It is the expression of the fact that if an element has an atomic number N the atomic weight of its lightest isotope cannot be less than $2N$. Worded as above, the ambiguity in the case of hydrogen is avoided. True atomic weights corresponding exactly to $2N$ are known in the majority of the lighter elements up to A.[36] Among the heavier elements the difference between the weight of the lightest isotope and the value $2N$ tends to increase with the atomic weight; in the cases of mercury it amounts to 37 units. The corresponding divergence of the *mean* atomic weights from the value $2N$ has of course been noticed from the beginning of the idea of atomic number.

The Number of Isotopes of an Element and their Range of Atomic Weight Appear to Have Definite Limits.—Since the atomic number only depends on the *net* positive charge in the nucleus there is no arithmetical reason why an element should not have any number of isotopes. So far the largest number determined with certainty is 6 in the case of krypton. It is possible that xenon has even more, but the majority of complex elements have only two each. The maximum difference between the lightest

and heaviest isotope of the same element so far determined is 8 units in the cases of krypton and xenon. The greatest proportional difference, calculated on the lighter weight, is recorded in the case of lithium, where it amounts to one-sixth. It is about one-tenth in the case of boron, neon, argon and krypton.

The Number of Electrons in the Nucleus Tends to be Even.—This rule expresses the fact that in the majority of cases even atomic number is associated with even atomic weight and odd with odd. If we consider the three groups of elements, the halogens, the inert gases and the alkali metals, this tendency is very strongly marked. Of the halogens—odd atomic numbers—all 6 (+1?) atomic weights are odd. Of the inert gases—even atomic numbers 13 (+2?) are even and 3 odd. Of the alkali metals—odd atomic numbers—7 are odd and 1 even. In the few known cases of elements of other groups the preponderance, though not so large, is still very marked and nitrogen is the only element yet discovered to consist entirely of atoms whose nuclei contain an odd number of electrons.

If we take the natural numbers 1 to 40, we find that those not represented by known atomic weights are 2, 3, 5, 8, 13, (17), (18), 21, (33), 34, (38). It is rather remarkable that these gaps, with the exception of the four in parenthesis, are represented by a simple mathematical series of which any term is the sum of the two previous terms.

In consequence of the whole number rule there is now no logical difficulty in regarding protons and electrons as the bricks out of which atoms have been constructed. An atom of atomic weight m is turned into one of atomic weight $m + 1$ by the addition of a proton plus an electron. If both enter the nucleus, the new element will be an isotope of the old one, for the nuclear charge has not been altered. On the other hand, if the proton alone enters the nucleus and the electron remains outside, an element of next higher atomic number will be formed. If both these new configurations are possible, they will represent elements of the same atomic weight but with different chemical properties. Such elements are called " isobares " and are actually known.

The case of the element hydrogen is unique; its atom appears to consist of a single proton as nucleus with one planetary electron. It is the only atom in which the nucleus is not composed of a number of protons packed exceedingly closely together.

Theory indicates that when such close packing takes place the effective mass will be reduced, so that when four protons are packed together with two electrons to form the helium nucleus this will have a weight rather less than four times that of the hydrogen nucleus, which is actually the case. It has long been known that the chemical atomic weight of hydrogen was greater than one-quarter of that of helium, but so long as fractional weights were general there was no particular need to explain this fact, nor could any definite conclusions be drawn from it. The results obtained by means of the mass-spectrograph remove all doubt on this point, and no matter whether the explanation is to be ascribed to packing or not, we may consider it absolutely certain that if hydrogen is transformed into helium a certain quantity of mass must be annihilated in the process. The cosmical importance of this conclusion is profound and the possibilities it opens for the future very remarkable, greater in fact than any suggested before by science in the whole history of the human race.

We know from Einstein's Theory of Relativity that mass and energy are interchangeable [36] and that in C.G.S. units a mass m at rest may be expressed as a quantity of energy mc^2, where c is the velocity of light. Even in the case of the smallest mass this energy is enormous. The loss of mass when a single helium nucleus is formed from free protons and electrons amounts in energy to that acquired by a charge e falling through a potential of nearly thirty million volts. If instead of considering single atoms we deal with quantities of matter in ordinary experience the figures for the energy become prodigious.

Take the case of one gramme atom of hydrogen, that is to say the quantity of hydrogen in 9 c.c. of water. If this is entirely transformed into helium the energy liberated will be

$$.0077 \times 9 \times 10^{20} = 6.93 \times 10^{18} \text{ ergs.}$$

Expressed in terms of heat this is 1.66×10^{11} calories or in terms of work 200,000 kilowatt hours. We have here at last a source of energy sufficient to account for the heat of the sun.[37] In this connection Eddington remarks that if only 10 per cent. of the total hydrogen on the sun were transformed into helium enough energy would be liberated to maintain its present radiation for a thousand million years.

Should the research worker of the future discover some means of releasing this energy in a form which could be employed, the human race will have at its command powers beyond the dreams of scientific fiction; but the remote possibility must always be considered that the energy once liberated will be completely uncontrollable and by its intense violence detonate all neighboring substances. In this event the whole of the hydrogen on the earth might be transformed at once and the success of the experiment published at large to the universe as a new star.

In considering the spectra of isotopes there is every reason to suppose that the light emitted by an atom will depend upon the movements of its planetary electrons, and therefore upon the force controlling these, that is, the nuclear charge. We therefore expect that the difference between the spectra of two isotopes will be extremely small since the nuclear charges are identical. This expectation is borne out in practice and the difference in wave-length has only been detected in the case of the isotopes of lead. Aronberg,[38] in 1917, discovered a shift of 0.0044 *A* between ordinary lead and a radio lead of atomic weight 206.3. This result has been confirmed the subsequent work of Merton,[39] who has recently measured a shift of 0.011 *A* in one of the lines of an extremely pure Carnotite lead as compared with the same line in ordinary lead. These shifts, though extremely minute, are however hundreds of times larger than the ones predicted by the simple application of the Bohr theory.

The artificial separation of the isotopes of non-radioactive elements in an exceedingly difficult operation; indeed, had it been otherwise they must have been discovered long ago. In the case of neon already described, which is a particularly favorable one, the extreme difference between the lightest and heaviest fractions amounted to 0.13 of a unit of atomic weight. Harkins,[40] using a somewhat similar diffusion method, has successfully separated the isotopic hydrochloric acids and obtained a shift of 0.055 of a unit. A beautiful method applicable to certain liquids has been developed by Bronsted and Hevesy.[41] This consists in allowing the liquid to evaporate at so low a temperature and pressure that none of the molecules escaping from its surface can ever return to it again, a concentration of the heavier constituent in the residue must then result. They first applied it to mercury and the latest separation achieved with the isotopes

of that element is indicated by the figure 0.99974 and 1.00023 for the densities of the lightest and heaviest fraction respectively, the normal density being taken as unity. In atomic weight this separation corresponds to a shift of 0.1 of a unit. They have also applied the same method to a solution of hydrochloric acid in water and obtained a change of atomic weight of about 0.02 of a unit.[42]

Several other methods of partial separation have been suggested but the only ones which have been successful in practice are those mentioned above. Complete separation can be achieved by means of positive ray analysis, but the quantities to be obtained in this way are too minute to be of the slightest practical importance. The fact that many of the most familiar elements prove to be mixtures of isotopes is of fundamental theoretical importance, but when we consider the extreme difficulties of their separation it seems very unlikely, unless some entirely new method is discovered, that the numerical constants of chemistry are likely to be affected seriously for some time to come.

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The Probability of Spontaneous Crystallization of Supercooled Liquids. C. N. HINSHELWOOD and H. HARTLEY. (*Phil. Mag.*, January, 1922.)—A hundred tubes containing solutions of the organic substances employed—salol, phenol, etc.—were given the same heat treatment, then placed in a thermostat and the number of tubes in which crystallization had occurred was observed at intervals. These investigators found that their results confirmed the conclusions of their predecessors "that the tendency of a supercooled liquid to crystallize depends upon the temperature to which it has been previously heated, on the number of times fusion and solidification have taken place, and on the length of time it has been maintained in the fused state." The tendency of salol to crystallize was found to be controlled by the highest temperature to which it had been heated without regard to whether crystallization had or had not taken place before the experiments were counted.

Some tubes lost progressively the tendency to crystallize. When they were exposed afresh to the air for a time and then resealed they were found to have regained the power.

These conclusions are drawn:

"1. In the case of these substances crystallization is provoked by the colloidal organic dust particles from the air.

"2. The activity of these particles diminishes in general as the result of heating or aging.

"3. Their effectiveness depends upon their radius. If this is equal to the radius of a small particle of the solid which should thermodynamically be in equilibrium with the supercooled liquid, then crystallization occurs at once. If the radius is less than this, the supercooled liquid has an average life depending on the discrepancy between the equilibrium radius and the radius of the particles present." G. F. S.

APLANATIC (OR CARTESIAN) OPTICAL SURFACES.*

BY

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HISTORICAL AND INTRODUCTORY.

LONG ago in his celebrated treatise on Geometry, Descartes described the focal properties of the curves which have been known ever since as the famous Ovals of Descartes, and whose equation is usually given in the form

$$r \pm \mu r' = a ;$$

where r, r' denote the distances of a point on the curve from two fixed foci and μ and a are constants. In his *Dioptrique* Descartes explained, as Huygens tells us, "the use of these lines in relation to refraction." Geometers have found much enjoyment in the study of these curves. In Williamson's "Differential Calculus," which used to be a famous text-book in England and in this country, an entire chapter is devoted to the subject of the "Cartesian Oval," and the curious reader will find there a very original and interesting discussion of its properties. The surface which is generated by the revolution of this curve around the straight line joining the foci as axis may be called here the Cartesian Surface. Theoretically, at least, a certain physical importance may be attached to this surface, because it represents the precise geometrical form which an optical surface should have in order to transmit radiant energy, either by refraction or by reflection, according to the known laws of these phenomena, accurately from one focus to the other. Accordingly, at least some allusion to this surface is made in nearly all the text-books and treatises on geometrical optics; although usually the subject is dismissed as having little practical significance, with perhaps an exception in favor of the parabolic reflector which is a special form of the surface for a mirror when one of the foci is infinitely far away. One of the earliest mathematical treatments of these optical surfaces in English is to be found in Sir John Herschel's article on "Light" in the *Encyclopædia metropolitana;* and

* Communicated by the Author.

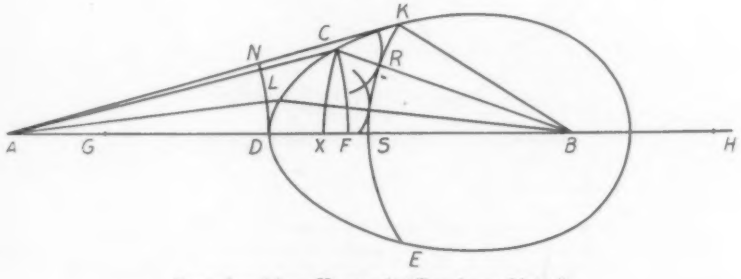
there are also two admirable chapters on the subject in Lloyd's "Treatise on Light and Vision" (London, 1831). In the appendix to the American edition of Brewster's "Optics" (Philadelphia, 1835), Professor Bache, the accomplished editor of this volume, has likewise devoted several pages to a discussion of the optical properties of these surfaces. But in more modern works, as, for example, Heath's "Geometrical Optics," not much space is given to this subject. Among German writers reference may be made to the treatises of Czapski, Lummer, Drude and von Rohr, in each of which there may be found a more or less extensive treatment of the so-called Cartesian surface.

The earlier writers on optics, for example, Coddington, Herschel, etc., spoke of an optical system as being "aplanatic," meaning thereby that it was so contrived that rays proceeding from a certain point on the optical axis issued from the system so as to be again reunited at a corresponding point on the axis. Modern writers, following Abbe, have restricted the meaning of this term so as to include also the fulfilment of the so-called "sine-condition" as an additional requisite of "aplanatism." But if the term is employed in its original wider sense (as will be done in this paper), the Cartesian surface may be referred to as an *Aplanatic Optical Surface.*

One of the most interesting portions of Huygens's "Traité de la lumière" (written in 1678 and communicated at that time to the Royal Academy of Science, but not published until twelve years later) is contained in the last chapter of that book, which in Thompson's English translation (London, 1912) is entitled "On the Figures of the Transparent Bodies Which Serve for Refraction and for Reflexion." Here Huygens proceeds to discover the form of "a surface CDE which shall reassemble at a point B, rays coming from another point A" (Fig. 1); the "summit of the surface" being at a "given point D in the straight line AB." With consummate skill he employs his famous method of construction of the enveloping wave-surface, and for the ordinary case of a refracting surface finds that "this curve is the same that Mr. Descartes has given in his Geometry, and which he calls the first of his Ovals." However, before setting out to solve this problem, Huygens himself says: "For though I do not see yet that there are means of making use of these figures, so far as relates to Refraction, not only because of the

difficulty of shaping the glasses of Telescopes with the requisite exactitude according to these figures, but also because there exists in refraction itself a property which hinders the perfect concurrence of the rays, as Mr. Newton has very well proved by experiment, I will yet not desist from relating the invention, since it offers itself, so to speak, of itself, and because it further confirms our theory of refraction, by the agreement which here is found between the refracted ray and the reflected ray. Besides, it may occur that someone in the future will discover in it utilities which at present are not seen." The opinion which Huygens here expresses of the impracticality of "aspherical lenses" has been prevalent ever since, and perhaps that is why

FIG. 1.



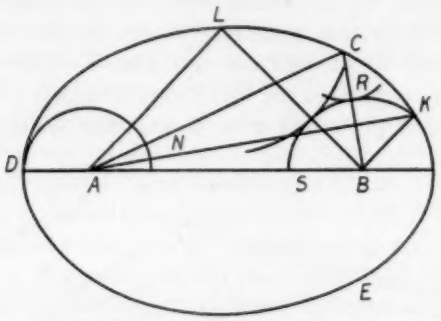
Reproduced from Huygens's "Treatise on Light."

they have received comparatively little attention. From time to time Abbe and others have revived the question in one form and another, and occasionally special types of aspherical optical surfaces have been computed and manufactured, as, for example, in the so-called "Katral" spectacle lens and in the illumination system of Gullstrand's ophthalmoscope; although let it be distinctly understood that the aspherical surfaces employed in these optical devices have nothing in common with the so-called Cartesian surfaces.

It would be both interesting and instructive to dwell more at length on Huygens's exceedingly ingenious geometrical analysis of the ovals of Descartes; but that would be beyond the scope and intention of this paper. Incidentally, of course, Huygens points out that when the rays are reflected the curve is an ellipse (Fig. 2) or a parabola (Fig. 3) in case the focus A is at an infinite distance. Moreover, in the case of refraction, when

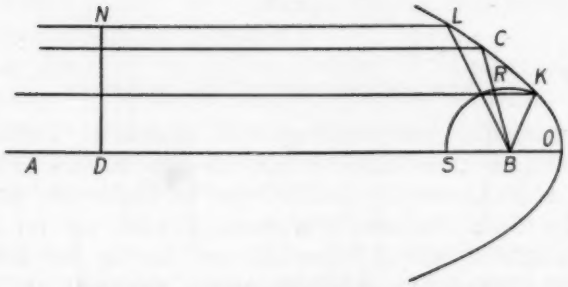
one of the foci is infinitely distant, the form of the curve is an ellipse (Fig. 4) when parallel rays in air are to be brought to a real focus in glass, or an hyperbola (Fig. 5) when parallel rays in glass are to be converged to a real focus in air. It may be mentioned here that Huygens was familiar with the so-called "aplanatic points"[1] of a spherical refracting surface, which he

FIG. 2.



Reproduced from Huygens's "Treatise on Light."

FIG. 3.



Reproduced from Huygens's "Treatise on Light."

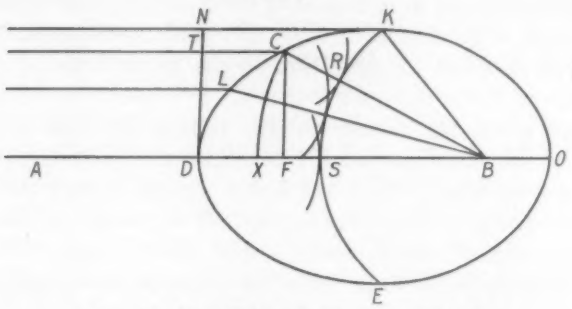
says were known to him "a long time ago"; from which it may be inferred that Huygens was the discoverer of these points to which attention was called again many years afterwards by Thomas Young in his "Lectures on Natural Philosophy" (London, 1807). Finally, Huygens explains clearly the construction of an aplanatic lens with at least one of its surfaces aspherical and designed so that for a given pair of axial points rays of

[1] See, for example, Southall's "Mirrors, Prisms and Lenses" (New York, 1918), p. 512.

light would be accurately refracted by the lens from one of these points to the other.

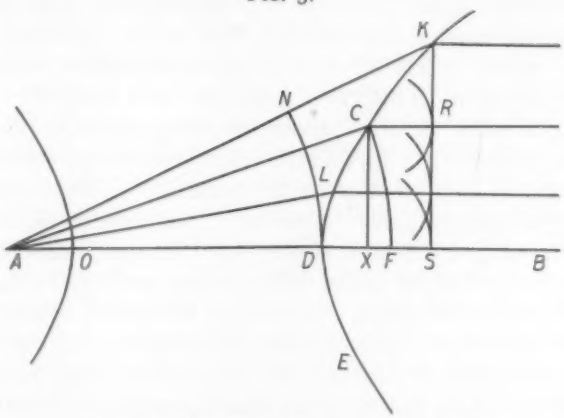
In this historical introduction it remains only to mention that the first published paper of James Clerk Maxwell was communicated to the Royal Society of Edinburgh by Prof. James Forbes,

FIG. 4.



Reproduced from Huygens's "Treatise on Light."

FIG. 5.



Reproduced from Huygens's "Treatise on Light."

in 1846, under the title, "On the Description of Oval Curves, and Those Having a Plurality of Foci"; wherein the author describes an ingenious method of constructing these curves with the aid of thread and pins similar to the ordinary mechanical way of drawing an ellipse. Maxwell was a schoolboy fourteen years of age when he wrote this paper. Neither he nor Professor Forbes was aware that Descartes' own process was precisely the

same in principle as that of Maxwell. A most interesting account of Maxwell's work may be found in his "Life" by Campbell and Garnett (London, 1882); and particularly in a "Note" appended to Chapter IV of this biography which contains a "series of propositions concerning the Ovals," with *facsimile* reproductions of young Maxwell's own drawings. The geometrical properties, as well as the optical significance of these curves, were apprehended by Maxwell with that rare ingenuity and mathematical acumen which was destined to place him in the front rank of the great physicists of the nineteenth century.

I have given the history of this subject in some little detail, because of this previous work I myself was not aware until after I had rediscovered the remarkable optical properties of the Cartesian ovals. Their story was of so much interest to me that I ventured to think others might share it also. Not much of this information is to be found in the usual text-books.

GEOMETRICAL AND ANALYTICAL INVESTIGATION.

Inasmuch as both Huygens and Maxwell (to recall only these two principal names) were, as has been stated, perfectly familiar with the optical effects of these so-called aplanatic surfaces, and since each of them in different ways and at a long interval apart has given a more or less exhaustive description of them from this point of view, any further contributions on the subject might almost seem to be a work of supererogation. The very fact that these earlier studies have fallen into comparative oblivion may serve as an argument to indicate that the practical applications are likely to be of slight value. This would perhaps be the opinion of competent authorities. However, in recent years notable advances have been made in the manufacture of optical glass both in this country and abroad; and it is now possible for opticians to obtain fairly good lenses, made of pressed glass, which are, of course, much cruder and cheaper than lenses with ground and highly polished surfaces, but which, nevertheless, are extraordinarily satisfactory and efficient for certain special limited purposes. Such lenses might be employed in flash lights, headlights on locomotives and automobiles, cheaper forms of searchlights, and condenser lenses in projection apparatus. It may, therefore, be worth while to direct attention anew to the possibility of using aspherical lenses which in Huygens's day,

at any rate, were attended with great difficulty of manufacturing. This idea was suggested to me by Dr. P. G. Nutting and led to my investigation of the matter.

The problem consists in ascertaining the form of the surface of revolution which will bend rays, either by refraction or by reflection, from a given point L on the axis to another given point L′ also on the axis; the position of the summit or vertex A being also assigned, where the straight line LL′ is to meet the surface. The foci designated here (and hereafter in this paper) by L and L′ may be "real" or "virtual," one or both, including therefore all possible cases. The step from L to A will be denoted by s, and that from A to L′ by c; thus $s = \mathrm{LA}$, $c = \mathrm{AL}'$, these steps along the axis being reckoned positive or negative according as they are in the same direction as the incident light or in the opposite direction, respectively. The relative index of refraction between the two media will be denoted by n; and hence if the absolute index of the first medium is put equal to unity, that of the second medium will be equal to n. When n is positive, and greater or less than unity, incident rays of light proceeding from the point L (or directed towards it, if L lies on the farther side of the surface) will be *refracted* to L′ (or as if they came from L′, when L′ lies on the nearer side of the surface towards the incident light); whereas in the special and unique case when $n = -1$, the light will be *reflected* in the same way from L to L′, and under these circumstances the surface in question will be an "aplanatic mirror." It goes without saying that it will be only necessary to determine the form of a meridian section of the surface of revolution. Moreover, since the central or paraxial rays constitute a portion of the wide-angle bundle of rays, the linear magnitudes denoted by s and c must be connected by the familiar abscissa-formula for the refraction (or reflection) of paraxial rays; which, expressed in terms of these symbols, will have here the following form:

$$\frac{n}{c} + \frac{1}{s} = \frac{n-1}{r}\,; \qquad (1)$$

where r denotes the radius of curvature of the meridian section of the surface of revolution at its vertex A, and is to be reckoned positive or negative according as this surface is convex or concave, respectively, towards the incident light.

Naturally, the axis of symmetry LL′ or the optical axis will

be taken as the abscissa-axis of a rectangular system of coördinates. The origin of this system may be conveniently chosen at one of the points L, L′ or A; suppose, therefore, we take the vertex A as origin. The problem may now be restated in terms of these symbols, thus: If a ray of light crossing the x-axis at the point L $(-s, 0)$ meets the surface at a point P (x, y) where it is deflected, by refraction or reflection as the case may be, and again crosses the axis, this time at the point L′ $(c, 0)$; it is required to find the locus of the incidence-point P.

The so-called ray-lengths are the steps, measured along the incident ray and the corresponding refracted (or reflected) ray, from the variable incidence-point P to the fixed points L, L′. These steps will be denoted here by l, l', thus: $l = \mathrm{PL}$, $l' = \mathrm{PL}'$; and are to be reckoned positive or negative according as they are in the same direction as the light goes along the ray or in the opposite direction, respectively. Evidently,

$$l^2 = y^2 + (s + x)^2 , \quad l'^2 = y^2 + (c - x)^2 \cdot \tag{2}$$

The optical length from L to L′ along the axis is equal to $(nc + s)$; whereas the optical length from L to L′ *via* a point P not on the axis is equal to $(nl' - l)$, the minus sign in this expression being due to the fact that both l and l' are measured *from* P, as above defined. If these two routes are traversed by the light in the same time, then

$$nl' - l = nc + s = \text{a constant} = a \quad \text{(say)}. \tag{3}$$

This is the fundamental focal equation and is equivalent to the usual form of the equation of the ovals of Descartes, as given in the very beginning of this paper.

If l, l' are eliminated from (3) by means of equations (2), the general equation in x, y of the meridian section of the aplanatic surface may be easily derived and can be written as follows:

$$\left\{ (n^2 - 1)(x^2 + y^2) - 2(n^2c + s)x \right\}^2 - 4n(nc + s)\left\{ (ns + c)(x^2 + y^2) - 2(n - 1)csx \right\} = 0. \tag{4}$$

In general, the locus represented by this integral, rational, algebraic equation of the fourth degree in x and y consists of two closed curves or ovals, one lying wholly within the other; but owing to the way in which the equation has been obtained in this fashion, that is, by squaring both sides in order to remove the radicals, one of these ovals will not be applicable to the particular optical problem which is under consideration. More-

over, the parameter n denoting the relative index of refraction actually has a very limited range of positive values, whereas it can have only one single negative value, *viz.*, the value $n=-1$.

If in equation (4) we put $y=0$, we shall obtain an equation of the fourth degree in x whose roots are the abscissæ of the four points A, B, C and D where the axis meets the two surfaces. Thus, we obtain:

$$x\left\{(n+1)x-2nc\right\}\left\{(n+1)x+2s\right\}\left\{(n-1)x-2(nc+s)\right\}=0,(y=0);$$

that is, $x=0$ (corresponding to the point A itself); $x=\frac{2nc}{n+1}=AB$ (which is the axial diameter of one of the two ovals); $x=-\frac{2s}{n+1}=$ AC; and $x=\frac{2(nc+s)}{n-1}=AD$, where CD is the axial diameter of the other oval.

The polar equation of the curve is easily obtained and affords another method of tracing its form. For this purpose it is convenient to take the focus L as the origin or pole and to denote by θ the slope-angle ALP which the radius vector LP makes with the axis; then since

$$l'^2=l^2+(s+c)^2+2l(s+c)\cos\theta, \qquad (5)$$

we obtain, by combining this equation with equation (3) so as to eliminate l', the polar equation of the curve in the following form:

$$(nc+s-\rho)^2=n^2\left\{\rho^2+(s+c)^2-2\rho(s+c)\cos\theta\right\}, \qquad (6)$$

where $\rho=-l$ denotes the radius vector LP. If in this equation we put $\theta=0$, we find

$$nc+s-\rho=\pm n(s+c-\rho),\ (\theta=0);$$

and hence for the two points A and B in which the axis meets the curve:

$$\rho=LA=s \text{ and } \rho=LB=s+\frac{2nc}{n+1}$$

Putting $\theta=\pi$, we find:

$$\rho=-LC=-\frac{n-1}{n+1}s \text{ and } \rho=-LD=-\left\{\frac{n+1}{n-1}(s+c)+c\right\}$$

which locates the vertices C and D of the other oval. However, this oval is not concerned with the optical problem of the transmission of radiant energy from L to L', which is here under

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The polar equation of the curve is easily obtained and affords another method of tracing its form. For this purpose it is convenient to take the focus L as the origin or pole and to denote by θ the slope-angle ALP which the radius vector LP makes with the axis; then since

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$$nc+s-\rho=\pm n(s+c-\rho),\quad(\theta=0);$$

and hence for the two points A and B in which the axis meets the curve:

$$\rho=LA=s \text{ and } \rho=LB=s+\frac{2nc}{n+1}$$

Putting $\theta=\pi$, we find:

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which locates the vertices C and D of the other oval. However, this oval is not concerned with the optical problem of the transmission of radiant energy from L to L′, which is here under

consideration. It is hardly necessary to add that the above results concerning the positions of the four vertices A, B, C and D are in agreement with those obtained previously.

The polar equation (6) may be written also as a quadratic in ρ in the following form:

$$(n^2-1)\rho^2+2\{(nc+s)-n^2(s+c)\cos\theta\}\rho + (n-1)\{(n+1)s+2nc\}s=0;$$

from which it appears immediately that, for given values of the parameters n, c and s, there will, in general, be two values of ρ corresponding to each value of θ.

It is easy to construct geometrically the positions of the two points P_1, P_2 on the locus corresponding to a given value of the $\angle L'LP=\theta$, as follows:

With L as centre, describe two concentric circles with radii equal to a and $\frac{a}{n}$, where $a=nc+s$. Through L draw a straight line inclined to LL′ at the given slope θ, and mark the point where this straight line meets the circle of radius a by the letter K. Draw the straight line KL′ meeting the other circle in the two points Q_1 and Q_2. Draw $L'P_1$ parallel to Q_1L meeting LK in P_1; and similarly, draw $L'P_2$ parallel to Q_2L meeting LK in P_2. Then P_1, P_2 are the two required points on the locus, as may easily be proved by an inspection of the diagram as above described, with the aid of the fundamental relation given by equation (3), which is here equivalent to saying that PK : PL′ = n : 1. If, for a given value of θ, the straight line KL′ does not meet the circle of radius a/n, then there are no points P_1, P_2 on the locus corresponding to this value of θ; or if the straight line KL′ is tangent to this circle, the two points P_1, P_2 are coincident, and in this case the given value of θ is the slope-angle of the outermost ray of the bundle emanating at L, and there will be no need of considering values of θ in excess of this limiting value.

After all the simplest way of studying the surface is by means of the fundamental focal property as expressed by equation (3). For example, differentiating this equation, we obtain:

$$dl - n\cdot dl' = 0 \; ; \qquad (7)$$

which shows that the function $(nl'-l)$ is a minimum (or a maximum). This is obviously equivalent to Fermat's "Principle of Least Time," and in fact we might have started with this law

and obtained by integration the equation which asserts the equivalence of the various optical routes from L to L′, as has been done by many writers on mathematical optics.

When the incident ray is parallel to the axis, then $l=-(s+x)$ and hence $dl=-dx$; therefore

$$n \cdot dl' = -dx , \quad (l = \infty) . \tag{8}$$

Or, again, if the ray after meeting the surface proceeds parallel to the axis, then $l'=c-x$, $dl'=-dx$; and hence

$$dl = -n \cdot dx , \quad (l' = \infty) . \tag{9}$$

If in equation (3) the constant a is equal to zero, then

$$nl' - l = 0, \quad nc + s = 0 ; \tag{10}$$

and for this special case the oval becomes a circle; that is, the aplanatic refracting surface is a sphere. This is the remarkable case which Huygens said had been known to him "a long time ago"; the pair of conjugate points L, L′ are identical with the so-called pair of aplanatic points J, J′ of a spherical refracting surface, which lie always both on the same side of the centre C of the surface ($CJ = n \cdot AC$, $CJ' = \frac{AC}{n}$), one of these points being a "real" focus while the other is a "virtual" focus.

In passing, it may be observed that when $n=-1$, equation (3) becomes $l+l'=$ a constant, which, properly interpreted, includes the well-known cases of elliptic, parabolic and hyperbolic mirrors with the radiant point situated at one of the geometrical foci of the conic section in question.

The case to which equation (8) applies, that is, the case when the incident rays are parallel to the axis, is of sufficient interest to be treated a little more in detail. When l becomes infinite, then

$$nl' + x = \text{constant}; \ (l = \infty);$$

and since $l'=c$ for $x=0$, the constant in this case will have the value nc; consequently,

$$nl' + x = nc, \quad (l = \infty). \tag{11}$$

Eliminating l' by means of (3), we obtain the Cartesian equation of the meridian section of the surface for this special case; which may be put in the following form:

$$y^2 = \frac{2(n-1)}{n} cx - \frac{n^2-1}{n^2} x^2, \quad (l = \infty). \tag{12}$$

This is the equation of a conic section in the form

$$y^2 = 4p\,e\,x + (e^2-1)\,x^2 ,$$

where $2p$ denotes the distance of the focus from the directrix and e denotes the eccentricity. Obviously, $e^2 = \frac{1}{n^2}$; and the radius of curvature at the origin is

$$\frac{n-1}{n} c = -\frac{n-1}{n} f' ,$$

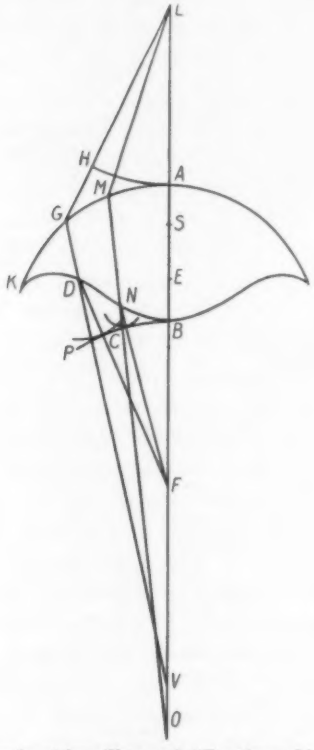
where $f' = F'A$ denotes the focal length in the second medium. (This is in agreement with the abscissa-relation for paraxial rays as given by formula (1); for if in this equation we put $s = \infty$, $c = -f'$ and solve for r, we obtain the same result as above.) The conic section will be an ellipse if $n > 1$ and an hyperbola if $n < 1$. Thus, incident rays parallel to the axis falling on a convex ellipsoidal surface of revolution and refracted from air to glass, or falling on a concave hyperboloidal surface and refracted from glass to air, will be converged to a real focus at a point L′ (or F′, let us designate it in this special case), such that $c = AL' = AF' = \frac{n}{n-1} r$, where $r = \frac{b^2}{a}$ denotes the radius of curvature of the ellipse and $r = -\frac{b^2}{a}$ denotes the radius of curvature of the hyperbola, each at the vertex A, and where $2a$ and $2b$ are the lengths of the two principal axes of the conic section in question. Although these two special cases were noted both by Huygens (see Figs. 4 and 5) and by Maxwell and are mentioned in some modern text-books and treatises, I think they are not generally known.[2] Dr. P. G. Nutting found this particular solution by an original and ingenious method which I had the privilege of seeing and which led me to study the general problem. It is interesting to remark that Kepler in his famous *Dioptrice* gives a proposition to show that the surface which will converge parallel rays in glass

[2] This article was already being printed when the author happened to see a letter in *Nature* (Feb. 16, 1922; vol. 109, pp. 205, 206) signed A. Mallock; in which, speaking of the advantage of aspherical lenses for microscopes, he alludes, by way of illustration, to the problem of finding "the form of a planoconvex lens which will convert spherical waves originating at [a point] O into a parallel beam;" and gives the following solution:

"Let the [required] convex surface of the lens cut the principal axis O X at A, and let the refractive index of the material be μ. The form of the surface is determined by the relation O A + μ A N = O P [where P is a point on the meridian section of the surface and N is the foot of the ordinate at P]. Elementary algebra shows that the curve P A is a hyperbola the asymptotes of which make angle $\tan^{-1}\sqrt{\mu^2-1}$ with the axis of the lens."

to a point in air is (as he says) "approximately hyperbolic"; [3] and the curious thing about it is that Kepler obtained this result,

FIG. 6.



Reproduced from Huygens's "Treatise on Light."

and indeed many of the properties of lenses and other optical contrivances, without knowing the true law of refraction.[4]

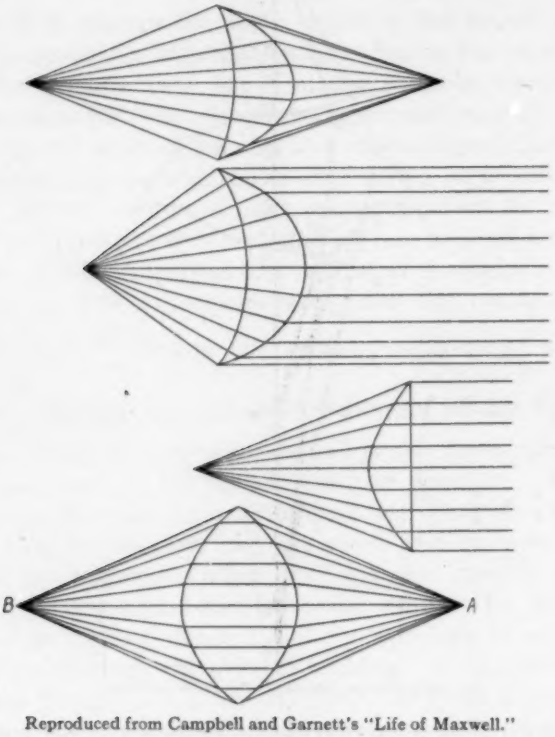
Huygens, in his "Treatise on Light," after having discussed the forms of the various ovals of Descartes, proceeds to show how the method of equal optical lengths which he employs "leads

[3] See Kepler's *Dioptrik*, Nr. 144, Ostwald's Klassiker der exakten Wissenschaften, (Leipzig, 1904), p. 27.

[4] See paper by R. A. Houstoun on "The Law of Refraction" in *Science Progress*, xvi (1922), pp. 397-407, in which Kepler's "one-constant" formula for refraction is given, with a table of Vitellio's measurements, showing how closely Kepler's formula agreed with the best experimental results available at that time.

without difficulty to the finding of the curves which one side of the glass lens requires when the other side is of a given figure, not only plane or spherical or made by one of the conic sections (which is the restriction with which Descartes proposed this problem, leaving the solution to those who should come after

FIG. 7.



Reproduced from Campbell and Garnett's "Life of Maxwell."

him), but generally any figure whatever; that is to say, one made by the revolution of any given curved line to which one must merely know how to draw the straight lines as tangents." The accompanying diagram (Fig. 6) which is reproduced from Huygens's work shows schematically the form of such an aplanatic lens, in which, the first surface AK being prescribed, "the outline BDK, which constitutes the other surface, shall be such that the path of the light from the point L to the surface AK, and from thence to the surface BDK, and from thence to the point F, shall be traversed everywhere in equal times, and in each

case in a time equal to that which the light employs to pass along the straight line LF of which the part AB is within the glass."

The diagram (Fig. 7), which is a *facsimile* reproduction of some of Maxwell's drawings as given in Campbell and Garnett's book, referred to above, shows certain obvious forms of aplanatic lenses. A glass lens designed to converge parallel rays to a real focus may be made, for example, either in the form of a convex meniscus lens with a spherical surface on the side next the focus (which is at the centre of the sphere) and an elliptic spheroidal surface on the other side, or in the form of a planoconvex lens with an hyperbolic spheroidal curved surface, as represented in the two middle diagrams of Fig. 7. Again, a symmetrical convex glass lens with hyperbolic spheroidal faces might be employed to transmit light-energy from a given point on one side of the lens to a point at an equal distance on the other side, as indicated in the lowest diagram in Fig. 7. The lens-thickness is immaterial, except in so far as the aperture of the system is dependent on this factor; but, as a matter of fact, for a large aperture the thickness will generally have to be rather excessive, it seems to me; which would probably preclude these forms from being used practically for flash-light systems, head-lights, condenser systems, etc.

In connection with the aplanatic optical surface, as defined by the fundamental focal relation (3) above, certain other general relations are also of sufficient interest to be mentioned here. Let $\theta = \angle ALP$ and $\theta' = \angle AL'P$ denote the slopes of a ray before and after refraction at the surface at P; and let ϕ denote the angle which the incidence-normal makes with the axis; and, finally, let α, α' denote the angles at P which the ray makes with this normal before and after refraction, so that according to the law of refraction:

$$n \cdot \sin \alpha' = \sin \alpha. \tag{13}$$

There are obvious geometrical relations between these angles, namely:

$$\varphi = \alpha - \theta = \alpha' - \theta'. \tag{14}$$

It should be remarked that the angles here indicated are all *acute angles* (or at any rate cannot have any greater absolute value than $\pi/2$) and are to be reckoned positive or negative according as the sense of rotation is counter-clockwise or the reverse, respec-

tively. The slope-angles denoted by θ, θ' may be more precisely defined in terms of the linear magnitudes, thus:

$$\sin\theta = -\frac{y}{l}, \cos\theta = -\frac{s+x}{l}$$
$$\sin\theta' = -\frac{y}{l'}, \cos\theta' = \frac{c-x}{l'}; \tag{15}$$

where x, y denote the coördinates of the point P referred to a rectangular system of axes with the origin at the vertex A.

Combining (13) and (14) so as to eliminate the angles of incidence and refraction, α and α', another perfectly general relation for the refraction of a ray at any surface of separation of two isotropic optical media may be obtained as follows:

$$\tan\varphi = \frac{\sin\theta - n\sin\theta'}{\cos\theta - n\cos\theta'} . \tag{16}$$

And if θ and θ' are eliminated from the right-hand side of this expression by means of formulæ (15), it becomes:

$$\tan\varphi = -\frac{(nl - l')y}{n(c-x)l + (s+x)l'} . \tag{17}$$

Moreover, since

$$\frac{dy}{dx} = -\cot\varphi , \tag{18}$$

the angle φ may be eliminated by combining (17) with (18), so that the preceding relation is equivalent to the following general condition:

$$nl\left\{\left(y\frac{dy}{dx}\right) - (c-x)\right\} = l'\left\{\left(y\frac{dy}{dx}\right) + (s+x)\right\}. \tag{19}$$

Now if the surface is to be aplanatic, the condition (3) must be imposed; hence, squaring both sides of the general equation (19) and eliminating l^2 by means of the special equation (3), we derive an equation of condition which must be satisfied by any aplanatic surface, namely:

$$A(s+x)^2 + B(s+x) + C = 0 , \tag{20}$$

where for brevity the symbols A, B and C have been introduced to denote the following expressions:

$$A = n^2z^2 - l'^2, \quad B = -2\left(y\frac{dy}{dx}\right)l'^2,$$
$$C = n^2y^2z^2 - l'^2\left(y\frac{dy}{dx}\right)^2, \tag{21}$$

and where

$$z = \left(y \frac{dy}{dx} \right) - (c - x) . \qquad (22)$$

Anyone who will take the trouble may convince himself that the condition (20) is satisfied by the general equation of the aplanatic surface as given by (4); but it is a laborious task, as the writer can testify.

One other general formula for the refraction of a ray at an optical surface of any form may be given here; which I think will be found useful. Suppose a ray of light is refracted at a point P on a surface separating two media whose absolute indices of refraction are n and n′; and let L designate the position of a point lying anywhere on the straight line which corresponds to the incident ray; similarly, let L′ designate the position of a point lying anywhere on the corresponding straight line determined by the refracted ray. Connect the two arbitrary points L, L′ by a straight line which meets the refracting surface in a point to be designated here by A. Evidently,

$$l' \cdot \sin\theta' = l \cdot \sin\theta , \qquad (23)$$

where the symbols θ and θ' denote the acute angles ALP and AL′P, respectively, and $l = \mathrm{PL}$, $l' = \mathrm{PL}'$.

Draw the normal to the surface at the point P, and let the point where this normal meets the straight line LL′ be designated by G. Put $g = \mathrm{GL}$, $g' = \mathrm{GL}'$ to denote the steps from G to L and to L′, respectively. Then

$$g : \mathrm{PG} = -\sin\alpha : \sin\theta , \quad g' : \mathrm{PG} = -\sin\alpha' : \sin\theta' ,$$

where α, α' denote the angles of incidence and refraction. Since $n \cdot \sin\alpha = n' \cdot \sin\alpha'$, the two proportions above may be combined into the following equation:

$$n'g' \cdot \sin\theta' = ng \cdot \sin\theta ;$$

and hence according to (23) above:

$$\frac{n'g'}{l'} = \frac{ng}{l} . \qquad (24)$$

This is an invariant relation for refraction which has long been known for the special case when the refracting surface is spherical;[5] but which has never to my knowledge been expressed as generally as in the form given here.

As an illustration of the use of formula (24), it may be

[5] See, for instance, Southall's "Mirrors, Prisms and Lenses" (New York, 1918), p. 518.

applied to derive the general equation of condition given by (20). Suppose the plane of incidence is taken as the plane of a system of rectangular axes with its origin at the point A where the straight line LL′ (x-axis) meets the refracting surface. Let the foot of the ordinate of the incidence-point P be designated by D, so that $x = \text{AD}$, $y = \text{DP}$, and $y\frac{dy}{dx} = \text{DG}$. Putting $m = \text{LD}$, we may write: $l^2 = y^2 + m^2$; and since

$$g = -\left(m + y\frac{dy}{dx}\right)$$

we obtain according to equation (24):

$$n'^2 g'^2 (y^2 + m^2) = n^2 l'^2 \left(m + y\frac{dy}{dx}\right)^2;$$

which may be arranged as a quadratic in m as follows:

$$(n'^2 g'^2 - n^2 l'^2)\, m^2 - 2n^2 l'^2 \left(y\frac{dy}{dx}\right) m$$

$$+ n'^2 g'^2 y^2 - n^2 l'^2 \left(y\frac{dy}{dx}\right)^2 = 0.$$

This equation is precisely equivalent to (20); as may be seen by writing $(s + x)$ in place of m, n in place of n′/n, and $-z$ in place of g'.

NEW YORK, N. Y.
MARCH 1, 1922.

The Centenary of the Discoveries of Ampère. (*Le Matin*, Nov. 25 and 26, 1921.)—On November 24, 1921, in the great amphitheater of the Sorbonne, France celebrated in a worthy manner the scientific achievements of her great son whose name has been given to the unit of electric current. The President of the Republic presided and addresses were made by Paul Appell, Paul Janet, Raynauld, Legouez and Boucherot. The tribute of the United State was given —and in the French language—by Dr. C. O. Mailloux, member and official delegate of The Franklin Institute, Philadelphia.

The four thousand persons present found interest in a table a metre square, on which was a complex of rods and wires. Though the humble product of a village locksmith it was none the less from the laboratory of Ampère to which so many of the applications of electricity can be traced.

The ceremonies were continued the next day at the Conservatoire des Arts et Metiers under the charge of Painlevé. He emphasized the discoveries made in the art of telegraphy in France in succession to the work of Ampère.

G. F. S.

PHOTOGRAPHY OF BULLETS IN FLIGHT.*

BY

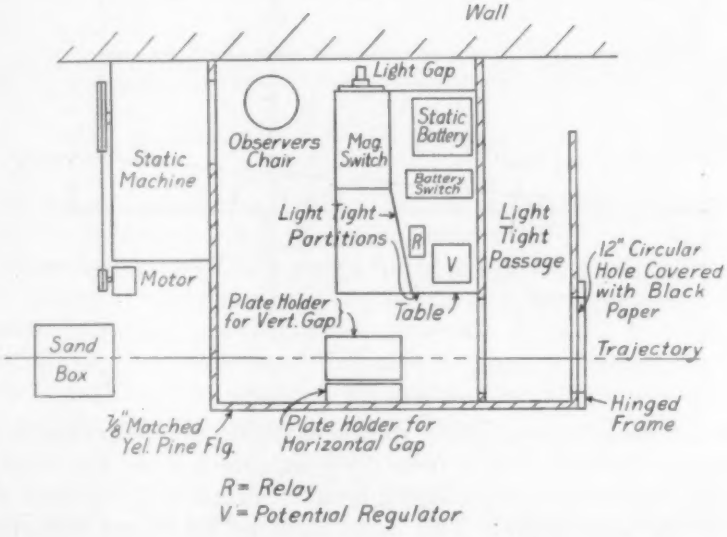
PHILIP P. QUAYLE.

Assistant Physicist, National Bureau of Standards.

THE study of the characteristics of a bullet in flight, together with its attending sound waves, is facilitated if instantaneous photographs of the phenomena are available. It is the purpose of this paper to outline the procedure by which such photographs are easily obtained.

The timing of mechanism employed in the apparatus to be

FIG. 1.



described in this paper is thought to be unique in that the bullet is not interfered with in any way and its photograph is obtained free from disfiguration by wires or other portions of the apparatus.

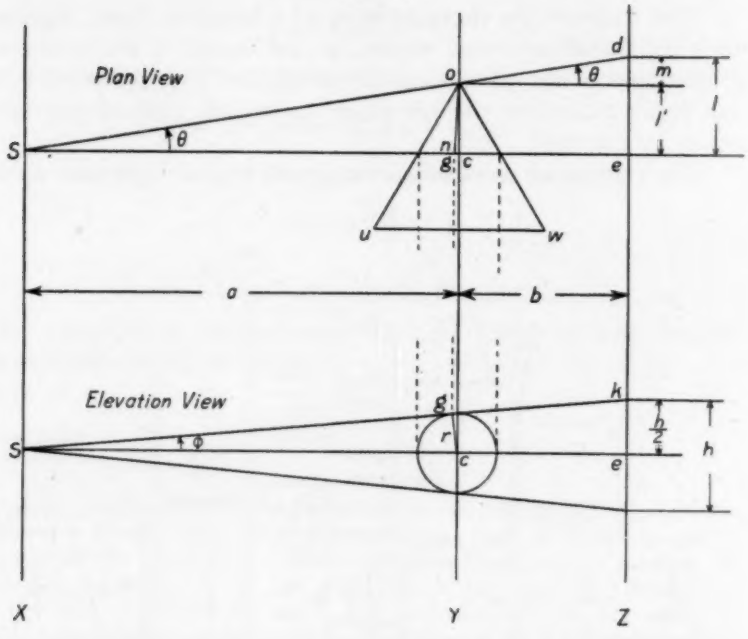
It is known that any mass moving through the atmosphere with a speed equal to or greater than that of sound causes a high compression of the air immediately in its front. This compression

* Communicated by Dr. E. A. Eckhardt.

radiates spherically from that point with the speed of sound. At the same time the mass from which the compressional wave is propagated is moving forward with its own speed, and if this is greater than that of sound, the resultant wave is not spherical but conical.

If such a wave is projected upon a photographic plate by means

FIG. 2.



of an instantaneous point source of light, the phenomenon is sharply depicted. Light from the point source in passing tangent to the wave must go through a layer of air much denser than the adjacent atmosphere. This dense layer of air causes refraction effects similar to those of a lens, and forms the image of the sound wave on the plate.

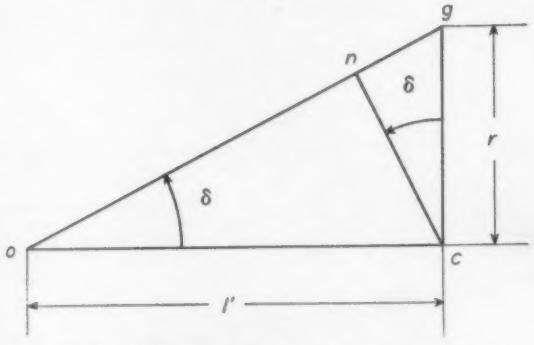
In the apparatus with which the photographs were obtained this sound wave controls the time of functioning of the spark, by means of an interrupter. The electrical circuit containing the spark gap had a natural frequency of oscillation of 4×10^{-6} seconds and was critically damped.

In general, the order of operation is as follows: The bullet, after leaving the rifle on its way to the small house in which the photographing is actually done, passes in front of an interrupter. The impinging of the sound wave upon the diaphragm of the interrupter automatically trips a switch inside of the apparatus house.

The bullet, while passing the photographic plate, is projected upon it by the spark controlled by this switch.

It is desirable to have the apparatus uniform in its operation

FIG. 3.

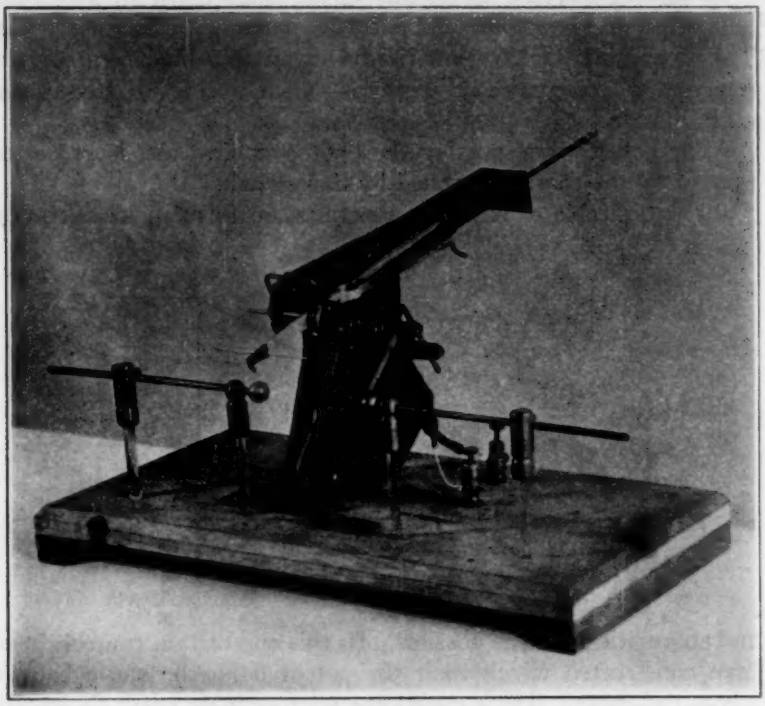


and automatic as far as possible. To this end certain controls have been constructed which limit the potential on the static battery where the charge, which causes the photographic spark, is stored. This static battery is charged by means of a large two-revolving-plate induction machine which is motor driven. A potential regulator was constructed and also a charging switch for the static battery. In operation the potential regulator was set off when the potential across the static battery was sufficiently high. The functioning of the regulator tripped the static battery switch, thus disconnecting the charging machine and short circuiting its terminals. The battery switch also turns on a signal light located outside the apparatus house as a signal to the rifle operator to fire. If the rifle is fired promptly at the signal, the operation of the apparatus is quite uniform.

ARRANGEMENT OF APPARATUS.

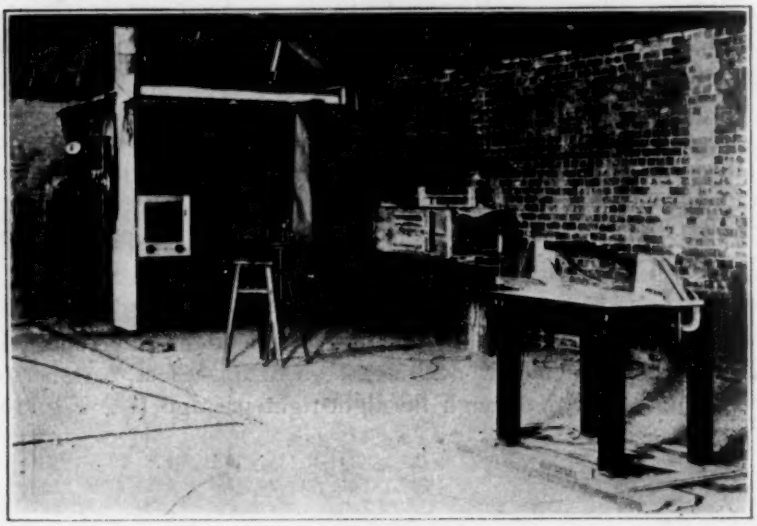
The bullet photography apparatus, except the interrupter, is entirely enclosed in a wooden house about 6 x 8 feet, entrance to which is obtained through the light-tight passage (see Fig. 1).

FIG. 4.



Magnetic switch which sets off the photographing spark.

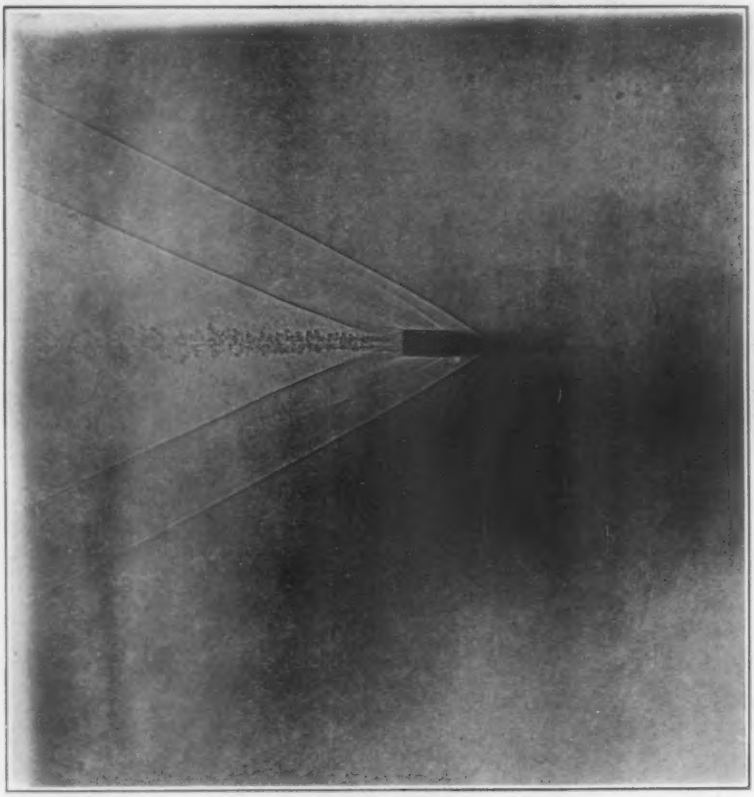
FIG. 5.



The laboratory in which the bullet photographs were made.

The magnetic switch which sets off the photographing spark is built into the frame holding the horizontal light gap, and this together with the large static battery, relay, battery switch, and potential regulator, is mounted on the table as indicated, and

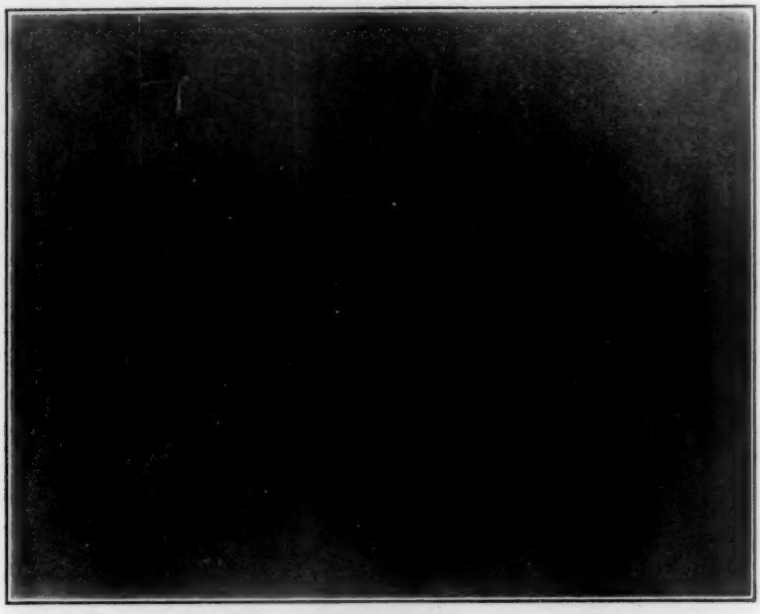
FIG. 6.



Modified Spitzer bullet, speed 3000 feet per second.

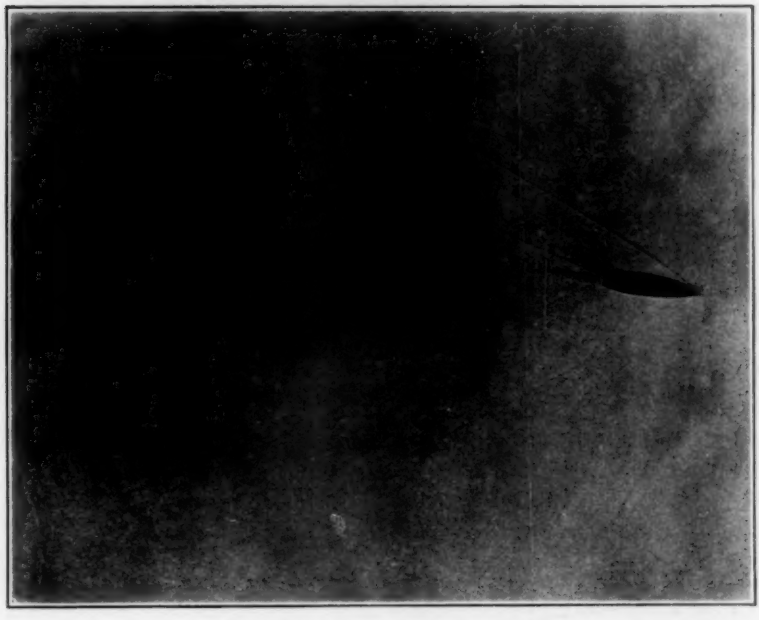
screened from the photographic plates by the light-tight partition. All the surfaces are painted a flat black. Directly in front of the horizontal light gap and distant from it 6.55 feet, a frame for the 11″ x 14″ plate holder is mounted. A similar arrangement in a vertical direction enables the apparatus to take two pictures of the same bullet, a plan and elevation. This is found to be very

FIG. 7.



30-calibre normal Spitzer bullet. Speed 2700 feet per second.

FIG. 8.



Boat-tailed bullet beginning to tumble.

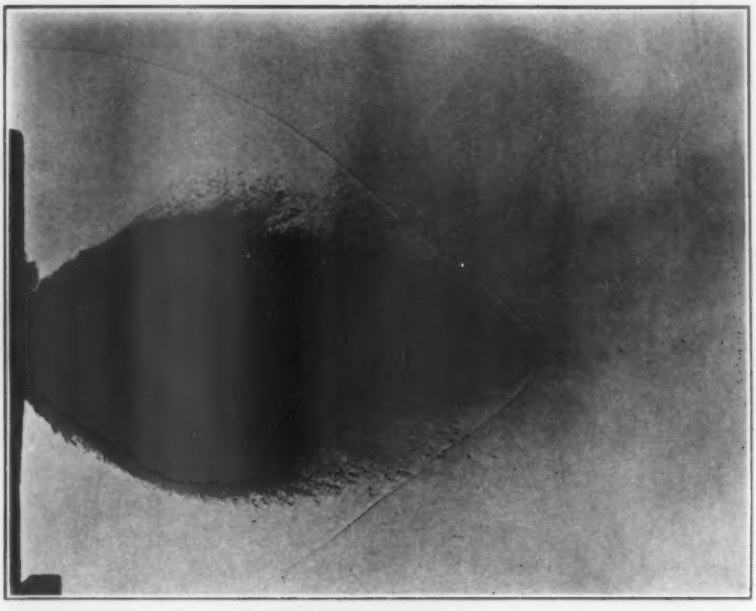
useful in investigating new types of ammunition which have a tendency to tumble.

THE OPERATION OF THE APPARATUS.

The operation of the apparatus is as follows:

The operator opens the magnetic switch and closes the battery switch. As he walks out he turns out the lights and draws the slides of the plate holders in use. He then loads the rifle, makes any correction necessary in the aiming and closes the motor switch

FIG. 9.



Flat-nose Spitzer bullet after going through 1-¾" plaster-of-Paris. Note that the jacket has been torn off and is following behind the bullet.

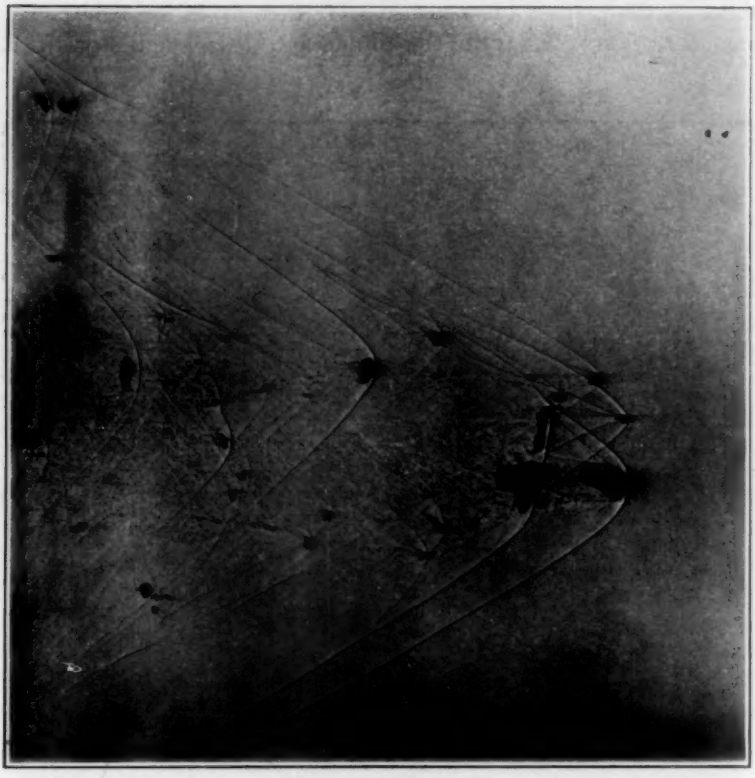
which is mounted on the rifle table. He takes up the slack in the rifle trigger, and as soon as the signal light comes on, fires. The motor switch is then opened, the slides replaced in the plate holders, and the plates are ready to be taken to the dark room for developing.

PRELIMINARY ADJUSTMENT.

In case nothing is known of the speed of the bullets to be investigated, a piece of ordinary wire screen (see Fig. 13) or paper is placed in the line of fire in front of the plate, and if its image

is punctured when the plate is developed, evidently the bullet had gone past the plate before the spark occurred. Since the lag of the apparatus is constant, the interrupter must be moved back from the plate.

FIG. 10.



Flat-nose Spitzer bullet after going through ¾" hard maple board. Note that the jacket has been torn off and is following behind the bullet.

VISUAL ADJUSTMENT.

When the adjustment of the apparatus is such that the bullet lies anywhere within the space illuminated by the spark it stands out sharply, and not only the bullet but also its shadow are sharply defined and visible to the most inexperienced observer. A strip of white cardboard when placed in the position normally occupied

by the plate makes an excellent background against which the bullet and its shadow contrast sharply.

When the visual method of adjustment is used an observer calls off the successive shots and the necessary changes in the interrupter distance are made until most of the bullets are seen in front of the plate as the spark illuminates it.

THE RELATION OF SPEED OF THE BULLET TO THE CONICAL SOUND WAVE.

Any bullet having a speed greater than that of sound propagates from both its nose and base a conical sound wave. These two waves are coaxial and the corresponding elements of their surfaces are parallel except in the immediate vicinity of the bullet; but due to the projection they diverge somewhat on the photographic plate.

At the apex of the cone the sound wave is travelling with the velocity of the bullet, while a short distance back from this point the wave slows down to the velocity of sound in air at the time of the experiment. The greater the velocity of the bullet the smaller the angle at the apex of the cone.

Since the distance of the light gap from the photographic plate is but six and one-half times the diameter of the largest portion of the conical sound wave photographed, a slight error would be made if the wave as photographed on the plate were assumed to faithfully represent the wave in air; for while the axis of the wave is projected in direct proportion to its length, it is evident that its radius is not. The radius of the projected element of the conical sound wave is not in a plane parallel to that of the photographic plate—hence the error.

It will be evident that if the true angle formed by the axis of the conical sound wave in air and that element of its surface which is projected upon the photographic plate were known, the velocity of the bullet producing the wave could be determined.

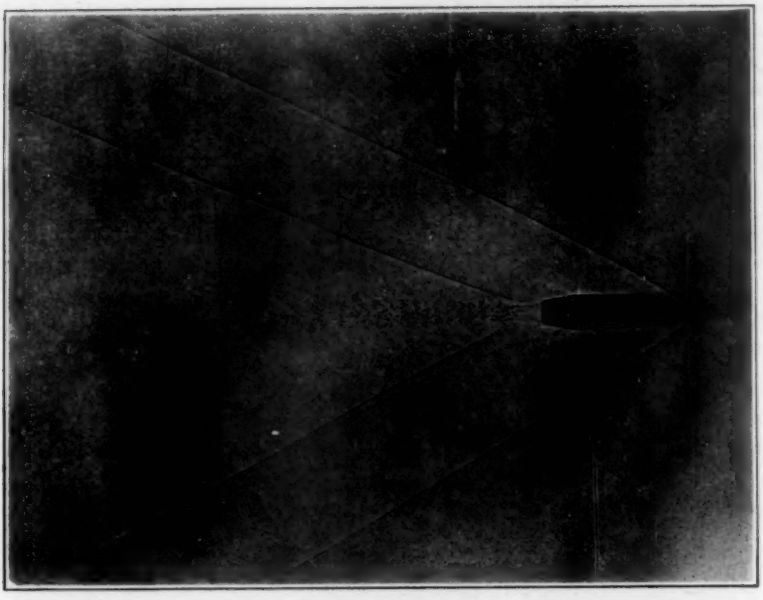
A formula giving these data will now be derived for a speed of sound equal to S feet/sec.

DISCUSSION OF THE PROJECTION OF THE CONICAL SOUND WAVE UPON THE PHOTOGRAPHIC PLATE.

Due to geometrical reasons, the image of the conical sound wave as projected upon the photographic plate is somewhat distorted.

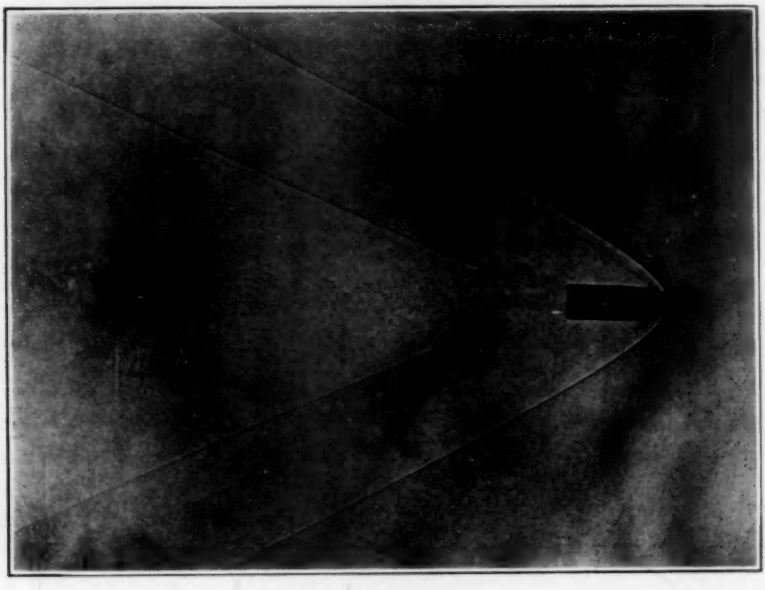
Referring to Fig. 2, Z is the plane of the photographic plate,

FIG. 11.



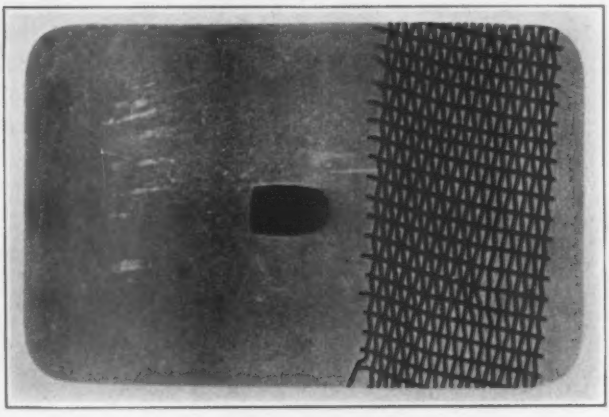
30-calibre boat-tailed bullet.

FIG. 12.



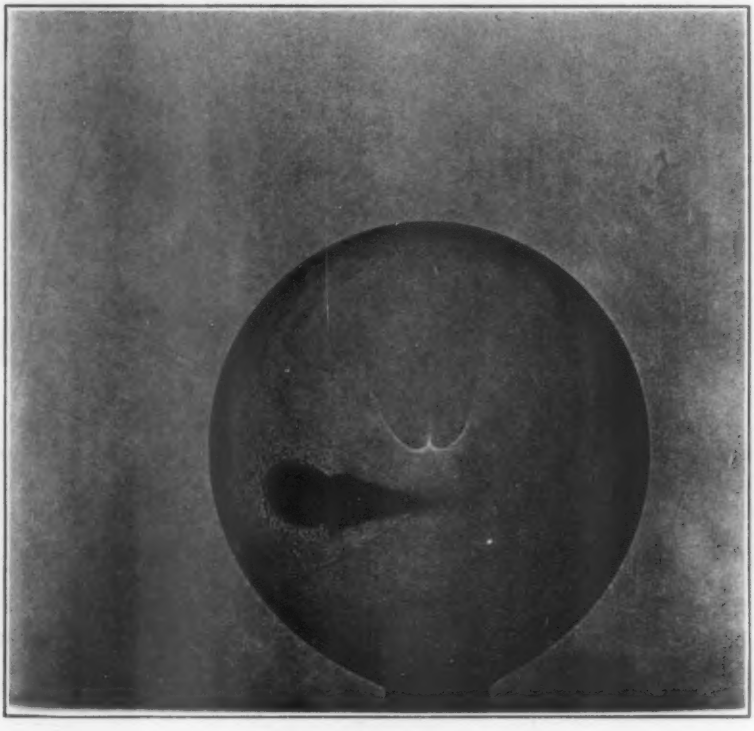
Flat-nose Spitzer bullet.

FIG. 13.



22-calibre bullet; no sound wave since the speed is approximately 900 feet per second which is less than that of sound.

FIG. 14.

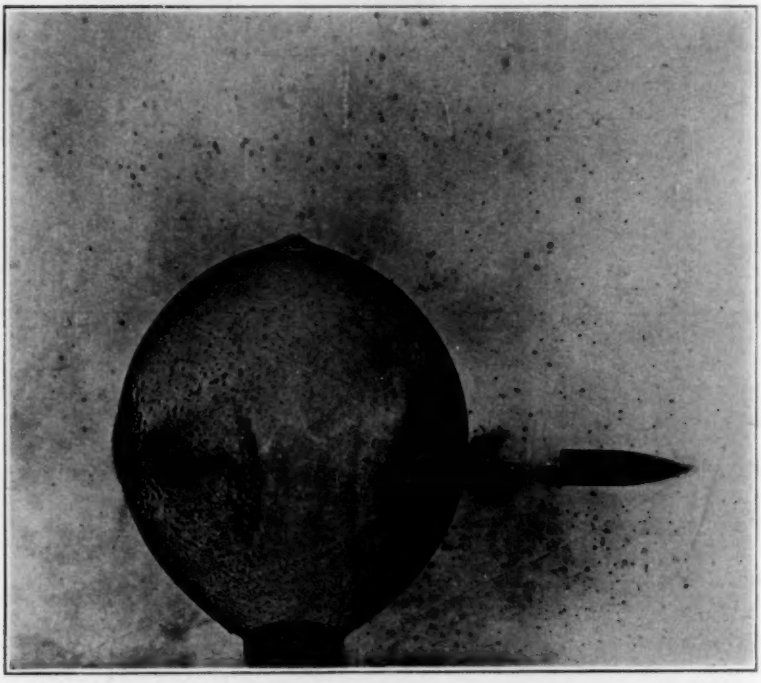


30-calibre Spitzer bullet entering soap bubble.

and Y and X are parallel planes passing through the trajectory and the light gap, respectively. From Fig. 2 it will be seen that l is the projection upon the photographic plate of that portion OC of the trajectory, which is also the axis of the conical sound wave $o\ u\ w$, see plan view, Fig. 2.

It is to be noted that l is measured from d, the intersection of

FIG. 13.



30-calibre Spitzer bullet after going through soap bubble.

the wave front produced as straight lines beyond the nose of the bullet, to the intersection of the plane of the base of the right circular cone with the photographic plate. At the point e, elevation view, $\frac{h}{2}$ is measured perpendicular to de; a is the distance of the plane of light gap X from that of the trajectory Y, and b is the distance of the plane of the trajectory from the photographic plate Z.

It is desired to find an expression for the true angle formed by the element *og* of the conical sound wave, and the axis *oc*. It will be evident that the work will be greatly simplified by taking the cutting plane perpendicular to the axis *oc*, its trace in the plan view being the line *sce*.

From Fig. 2, it will be seen that:

$$m = b \tan \theta \text{ and } l' = l - m \therefore l' = l - b \tan \theta$$

$$\theta = \tan^{-1} \frac{l}{a+b} \qquad \phi = \tan^{-1} \frac{h}{2(a+b)}$$

$$r = a \sin \phi = a \sin \tan^{-1} \frac{h}{2(a+b)}$$

It will be seen that l' and r are respectively the base and altitude of the triangle formed by the axis *oc* of the conical sound wave and that element *og* of its surface which is contained in the tangent plane passing through the light gap *s*. Let this angle be δ (see Fig. 3), then

$$\tan \delta = \frac{r}{l'}$$

$$\text{or } \tan \delta = \frac{a \sin \phi}{l - b \tan \theta}$$

$$= \frac{a \sin \tan^{-1} \dfrac{h}{2(a+b)}}{l - b \dfrac{l}{a+b}}$$

Since l' is proportional to the velocity of the bullet and *nc* to the velocity of the sound in air at the time the plate is exposed, it follows that the velocity of the bullet in terms of the speed of sound is given by

$$V = \frac{l'}{n\,c} S$$

Where S = the speed of sound in air at the time the plate is exposed.

From Fig. 3 it will be seen that $nc = l' \sin \delta$, supplying this value in the velocity equation gives

$$V = \frac{l'}{l' \sin \delta} S = \frac{S}{\sin \delta}$$

In order to obtain values of V in terms of S for different values of $\sin \delta$ it is first necessary to solve

$$\tan \delta = \frac{a \sin \tan^{-1} \dfrac{h}{2(a+b)}}{l - b \dfrac{l}{a+b}}$$

for delta. It is to be noted that a and b are held constant for any given set up.

By supplying in the formula

$$V = \frac{S}{\sin \tan^{-1}\left[\dfrac{a \sin \tan^{-1} \dfrac{h}{2(a+b)}}{l - b\,\dfrac{l}{a+b}}\right]}$$

a table could be computed for different vaules of h while l is held constant. It is to be noted that for every different l a new table must be computed if the above formula is used. As previously pointed out, l must be measured from the point of the cone to the intersection of the plane of the base of the right circular cone with the photographic plate.

In the set-up with which the photographs were obtained the light gap is directly opposite the centre of the plate. Therefore, when the bullet is photographed in the centre of the plate the light gap lies on the perpendicular to the sound wave axis erected at its apex.

In this position there is no error of projection, for the elements of the conical sound wave which are projected on the plate are then both contained in a plane which passes through the wave axis and is parallel to the plate.

The utilization of the bow wave for triggering the spark and the type of springs used in the magnetic switch were suggested by Dr. D. C. Miller, in charge of the Physical Laboratory at Case School of Applied Science, where the work was done.

The Application of the Ultra-micrometer to the Measurement of Small Increments of Temperature. W. SUCKSMITH. (*Phil. Mag.*, January, 1922.)—Two oscillating valve circuits are so joined up that a variation in the capacity of a condenser will cause a change in the number of beats per second produced by the interference of a note from a tuning fork with the heterodyne note in a telephone receiver connected in the circuit. The condenser consists of two parallel metal plates, one of which is carried by a metal rod whose change of temperature is to be determined. With a knowledge of the constants of the apparatus it is possible to detect a change of temperature of one-sixteen thousandth of a degree Centigrade, or a change of length of one two-hundred millionth of an inch. The expansion coefficient of copper was measured by raising the temperature a single degree. G. F. S.

A PRECISION ROTATING SECTOR: APERTURE VARIABLE AND MEASURABLE WHILE IN MOTION.*

BY

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THE validity of Talbot's law has been so amply established by the work of Brodhun, Hyde and others, that the rotating sector has attained widespread use in photometry. This is not surprising in view of the fact that the simple rotating sector, as a means for varying the intensity of light, possesses the following advantages:

1. It is an "absolute" instrument.

2. It is non-selective and is, hence, equally applicable to work in the infra-red, visible and ultra-violet spectrum.

3. It does not modify the character of the incident light—as is the case with polarization photometers.

4. It possesses a large field of view that is uniformly illuminated.

5. It makes for compactness of construction—in contrast to the application of the "inverse-square law."

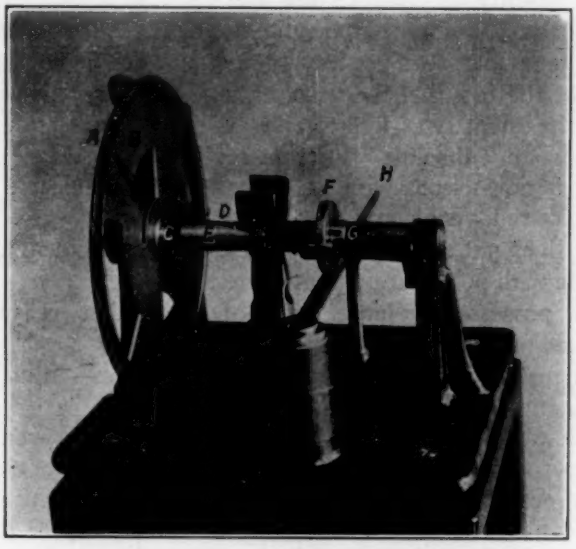
In view of these cardinal virtues it is not to be wondered at that numerous attempts have been made to transform the simple rotating sector into an ideal instrument by making the angular opening both variable and accurately measurable while the sector is in motion. Noteworthy attempts in these directions have been made by Abney, Brodhun, Keuffel and others. While the sector of Brodhun swings the light-beam over a stationary and variable aperture by means of glass rhombs (thus sacrificing non-selectivity) the others, as a rule, have a lever whose motion is transmitted to the sectors by means of clutches, gears, etc. The angular opening is read off a scale over which the lever moves. In consequence of unavoidable "lost motion," the scale reading thus obtained does not yield the angular opening with precision. Recognizing this difficulty, some investigators have chosen the course of stopping the sector in order to read the angular opening from the scale engraved on the sectors themselves. Due to inertia effects which

* Communicated by Joseph S. Ames, Ph.D., Associate Editor.

assert themselves when the sectors are stopped, the angular opening as read is not that which existed while the sector was in motion—hence this mode of procedure also does not yield accurate results.

The instrument developed by the writer overcomes these difficulties. By means of a simple stroboscopic device, the angular opening is read off the scale engraved on the sectors themselves.

FIG. 1.

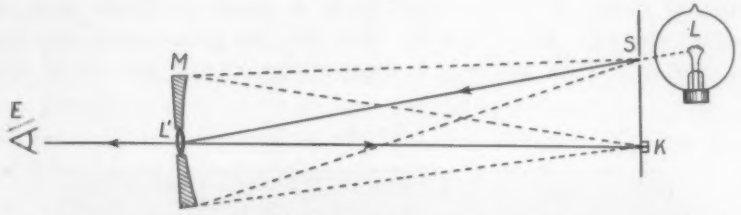


In addition, the sectors are so cut that two adjacent fields of view are presented. As the sectors are shifted, one field increases while the other decreases in brightness. In extreme positions, one field is black while the other is at maximum brightness—conditions are reversed as the sectors rotate, relatively through 90°. The shifting of the sectors is accomplished by well-known means.

The general construction of the instrument is shown in Fig. 1. Here, *A* and *B* are the two sectored discs; *A* is rigidly attached to the central shaft while *B* is attached to a sleeve *C*. This bears a pin which moves in the spiral slot *D* cut in a second sleeve *E*. Longitudinal motion of this sleeve is established by means of a clutch at *F*. Rotation of the sleeve *E* relative to that of the main shaft is prevented by a slot-and-pin arrangement at *G*. In order

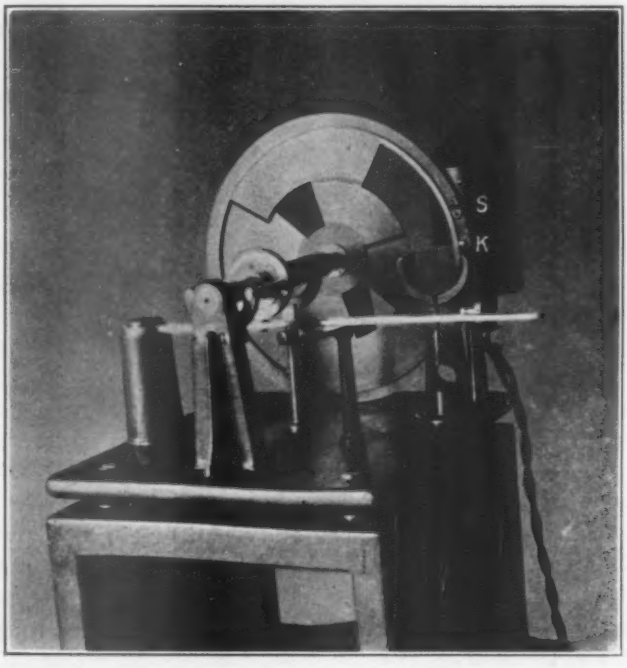
to bring about a relative motion of the discs A and B it is only necessary to move the clutch lever H back and forth. This motion

Fig. 2.



is quite free even when the speed of rotation exceeds 2500 R.P.M. and adjustments of the sector opening may be made by the most

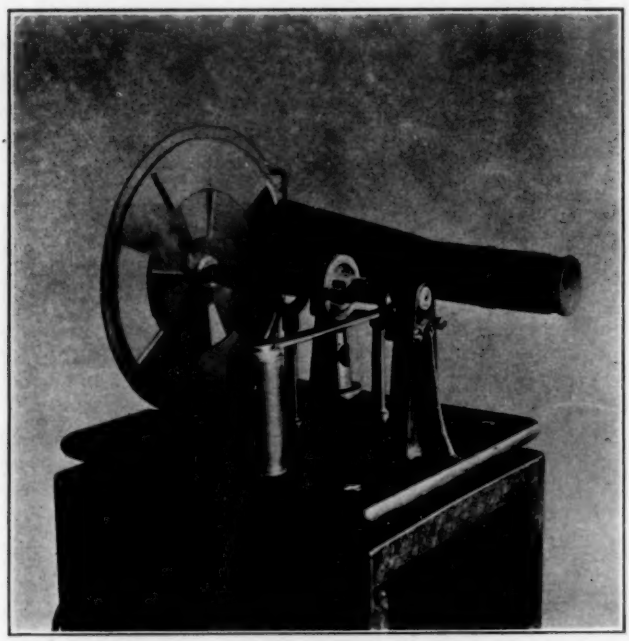
Fig. 3.



minute steps. Power is applied by means of a small motor (not shown) connected by a small belt to a pulley mounted on the end of the main shaft.

The simple stroboscopic device used to read off the relative positions of the two sectors is shown in Fig. 2. A narrow slit S is cut in the sector about three-quarters of an inch above the fiducial mark K. The light from a small tungsten lamp L passes through this slit and falls on the gold-coated concave mirror M, which projects a sharp image of the slit on K and

FIG. 4.



on the portion of the graduated scale immediately adjacent. The scale and marker K are polished and lacquered so as to reflect a large amount of the incident light back normally through the small lens L' mounted in a hole cut into the centre of the concave mirror. This lens is of such focal length as to present to the eye at E a magnified image of the scale. When the sector is set in motion, the scale is seen as clearly as though it were at rest—in fact, the most minute details such as scratches, etc., are depicted with astonishing clearness. For sharp delineation it is absolutely essential that the slit image be focussed sharply on the scale and

that the slit be as narrow as is consistent with obtaining light of sufficient intensity. The disposition of S and K are shown in Fig. 3 (the tube containing the mirror has been removed). If the sectors be rotated very slowly by hand, it is found that the slit image moves over the scale in a direction opposite to that of the scale. Since the relative velocity of image and scale is twice the peripheral velocity, it is observed that the stroboscopic image of the scale is contracted to one-half of its true width. In spite of this, it is easy to read the scale to 0.1 of a division.

It is, of course, clear that this stroboscopic device may be employed wherever the relative motion of rotating discs is to be measured. Experiments with motors, dynamos, phosphoroscopes, etc., offer opportunities for such applications. A consideration of the possibilities of various alternative schemes involving the use of periodic electrical contacts, sparks, telephones, etc., has convinced the writer that they are all inferior in accuracy to the direct reading of the scale engraved on the rotating discs themselves. The complete instrument with lamp-house and tube containing the mirror and lens system is shown in Fig. 4.

The application of this instrument to photometry, spectrophotometry and colorimetry are obvious. Altogether the instrument leaves but little to be desired.

The effects of "lost motion," "play," etc., are entirely eliminated, hence this instrument is justly termed "precision rotating sector."

In conclusion the writer wishes to thank Mr. J. P. Gaffney, University mechanician, for the great skill and the painstaking care which he has brought to bear on the construction of the instrument.

March, 1922.

Digestion of Raw Egg White.—Mary S. Rose and Grace MacLeod, of Columbia University (*Jour. Biol. Chem.*, 1922, 1, 83-88), have studied the digestion of the protein of white of egg by man. The whites of from 10 to 12 eggs were consumed daily by each of ten subjects. The coefficient of digestion of cooked whites was 85.9 per cent., of unbeaten raw whites 76.6 per cent., of partly beaten and partly unbeaten raw whites 80.7 per cent., and of raw whites beaten light 83.8 per cent.; the average coefficient for raw whites of all types was 80.4 per cent. The conclusion is drawn that it is unnecessary to emphasize the difference between raw and cooked eggs, provided the raw eggs be beaten. J. S. H.

Some Facts Bearing on the Structure of Atoms, Particularly of the Helium Atom. R. A. MILLIKAN. (*Phys. Rev.*, Dec., 1921.)—Professor Millikan has developed a wonderful method by which he can measure the electric charge on the fragment of an atom to the same degree of accuracy with which the census enumerates the number of people in the United States. When he applies this method to the atoms that have suffered bombardment by alpha rays coming from a bit of polonium he can tell exactly how many electrons have been knocked off, because each detached electron has carried away a definite and known quantity of negative electricity. When oxygen, nitrogen or carbon with their few electrons or iodine or mercury with their many electrons are subjected to assaults of the alpha particles, it is found that a single such particle practically never detaches more than one electron. "This, in itself, throws a certain light on atomic structure, for it shows conclusively that the electrons within an atom act quite independently of one another. They are certainly not in rings, of say four or eight or any other number which become unstable when one of their number is removed, or which can in general be shattered as a whole."

With helium it is different. The result is thus picturesquely phrased, "The alpha particle shooting at random through the helium atom at its maximum ionizing speed gets both electrons every sixth shot in which it gets anything." Such a fact does not determine the structure of the helium atom but it does make certain structures impossible. One such structure thus ruled out is the original Bohr arrangement according to which the helium atom has its two electrons at opposite ends of a diameter. Another is that suggested by Sommerfeld in which one electron rotates close to the nucleus while the other is in an orbit much farther away.

To kill the traditional two birds with one stone it is necessary that they be close together at the time of their demise. Similar reasoning leads to the conclusion that the two electrons which get knocked off once out of six must be close together in the atom a considerable part of the time. This is in consonance with the recent arrangements of Landé and of Bohr, according to which the electrons move in roughly equal orbits whose planes are inclined to each other. "Bohr may also find in it support for his recent contention that the outer shell of heavy atoms possesses few electrons instead of many as postulated in most of the discussions of the "static atom."

This article is from the Norman Bridge Laboratory of Physics, Pasadena, California, and may perhaps be regarded as the first fruits of Colonel Millikan's activities in his new surroundings.

G. F. S.

ON A METHOD OF CALCULATING FLUIDITY, SURFACE TENSION AND REACTION (EQUILIBRIUM) PRESSURE.*

BY

HENRY JERMAIN MAUDE CREIGHTON, D.Sc.

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SOME years ago it was shown by Ramsay and Young [1] that for any pair of closely related substances—such as methyl acetate and ethyl acetate, or propyl propionate and propyl butyrate—the ratio of the absolute temperatures (T) corresponding to equal vapour pressures is constant, *i.e.*, $T'_A/T'_B = T_A/T_B =$ a constant. For substances not closely related, it was found that the relation was less simple, but that it might be expressed by the equation $R' = R + c(T'_B - T_B)$, where R' is the ratio of the absolute temperatures of the two substances corresponding to any vapour pressure, the same for both; R is the ratio of the absolute temperatures at any other vapour pressure, again the same for both; and T'_B and T_B are the temperatures of one of the substances corresponding to the two vapour pressures. This relationship, which was tested by Ramsay and Young for 23 pairs of substances, has been found to hold up to the critical point. The method has been employed by Ramsay and Travers [2] to calculate the vapour pressures of the inert gases, argon, krypton and xenon, at various temperatures.

At the suggestion of Ramsay, Findlay [3] showed that a precisely similar equation to that of Ramsay and Young connects the absolute temperatures at which two substances have equal solubilities, and also the absolute temperatures at which two chemical equilibria have equal equilibrium constants.

The writer has found that the absolute temperatures at which two substances have the same value for certain other physical constants are related by an expression having the same form as the Ramsay and Young vapour pressure equation. In this paper the relations between the absolute temperature and fluidity, the

* Communicated by Professor Creighton.

[1] *Phil. Mag.* (5), **20**, 515 (1885); **21**, 33 (1886).

[2] *Phil. Trans.*, A., **197**, 47 (1901).

[3] *Proc. Roy. Soc.*, **69**, 471 (1902).

absolute temperature and surface tension, and the absolute temperature and reaction pressure (equilibrium pressure) are presented briefly.

The constant, c, in the equation $R' = R + c(T'_B - T_B)$, where R' and R are the ratios of the absolute temperatures of two substances, A and B, corresponding to two values of the physical constant, is determined as follows: Values of the ratio $R' = T'_A/T_B$, corresponding to a number of different values of

TABLE I.

Calculation of the Fluidity of Octane (A) from the Fluidity of Benzene (B).

$c = 0.000354$.

Fluidity ϕ	Observed Absolute Temperature†.		Ratio of Observed Temperatures T'_A/T'_B	Calculated Absolute Temperature T'_A	Calculated Fluidity of Octane ϕ calcd.	Difference ϕ calcd.—ϕ
	Octane[4] T'_A	Benzene[5] T'_B				
142.2	273.1	287.6	0.9497	273.4	142.6	+0.4
163.2	283.1	296.8	0.9540*	283.1	163.2	0.0
174.8	288.1	301.5	0.9557	288.1	174.8	0.0
185.9	293.1	306.1	0.9576	293.0	185.7	—0.2
197.4	298.1	310.8	0.9591	298.0	197.2	—0.2
209.9	303.1	315.5	0.9605	303.2	209.2	+0.2
233.7	313.1	324.9	0.9637	313.1	233.7	0.0
259.4	323.1	334.1	0.9673	322.8	258.8	—0.6
286.1	333.1	343.4	0.9701	333.2	286.3	+0.2

† In most of the examples which follow, the T'_A values are direct observations, while the T'_B values have been obtained by interpolation of experimental data.

the physical constant, are plotted against the absolute temperatures, T'_B, of one of the substances. A straight line is then drawn through the series of points obtained and "smoothed ratios" read off, corresponding to the temperatures T'_B. The values of the smoothed ratios are substituted in the equation $R' = R + c(T'_B - T_B)$ and the equation solved for c.

The success with which the fluidity (ϕ) of liquids can be calculated by means of the equation, $R' = R + c(T'_B - T_B)$, is illustrated by the examples given in Tables I and II. The former table gives in detail data obtained with octane (A) and benzene (B), while the latter, summarizing the results obtained, using propyl iodide (A) and perchlorethylene (B) and water (B) with each of several liquids (A), gives the "observed" and "calcu-

[4] Landolt-Börnstein, "Physikalisch-Chemische Tabellen," p. 78, Berlin, 1912.

[5] *Ibid.*, p. 80.

lated" absolute temperatures at which liquid A has a definite fluidity. In these and the following tables the asterisk denotes the temperature ratio R (or the absolute temperatures T_A and T_B, where $T_A/T_B = R$), from which the other temperature ratios (R') and the corresponding absolute temperatures (T'_A) have been calculated.

It will be observed that the calculated and observed values in these tables agree very well, except in the case of mercury and water. This lack of agreement is partly due, at least, to the flatness of the fluidity-temperature curve of mercury, in consequence of which a small error in the fluidity gives a comparatively large error in the calculated value of the absolute temperature. Above $T'_A = 427.1°$, the difference between the observed and calculated absolute temperatures increases rapidly with rise in temperature. This may well be due to inaccuracies in the experimental data, since the values for the viscosity of mercury at high temperatures, as determined by Koch[6] and by Plüss,[7] differ considerably. It should be noted that much better agreement over a greater range has been obtained with fluidity values based on Koch's viscosity mesaurements than with those based on Plüss' determinations.

A number of other pairs of liquids that have been studied are given in Table III. With one exception, the agreement between the observed and calculated values of T'_A, over the temperature interval studied, is as close as that given in Tables I and II. Indeed, in most cases, the agreement between the observed and calculated fluidity is closer than the values determined by different investigators. The pair of liquids, ethyl alcohol: water, does not follow the equation $R' = R + c(T'_B - T_B)$ even approximately. With these two liquids the temperature ratio T'_A/T'_B, when plotted against T'_B, instead of changing linearly, at first increases rapidly as T'_B increases, then more slowly and finally decreases. With aniline and water the agreement is good over the fluidity range given in the table, but the difference between the observed and calculated temperatures increases rapidly above the upper limit given for the fluidity. This is hardly surprising, as at higher temperatures values for the viscosity of aniline determined by different observers vary by 20 per cent. or more (*e.g.*, the values

[6] *Wied. Ann.*, **14**, 1 (1881).
[7] *Z. anorg. Chem.*, **93**, 1 (1915).

TABLE II.

Calculation of Absolute Temperatures Corresponding to Given Fluidities.

Methyl alcohol[8] (A) and Water[9] (B). $c = 0.000568$				Acetic acid[10] (A) and Water[9] (B). $c = 0.00217$				Mercury[11] (A) and Water[9] (B). $c = 0.0321$				Phenol [12] (A) and Water[9] (B). $c = 0.00118$				Propyl Iodide [13] (A) and Perchlorethylene [14](B). $c = 0.000174$			
T'_B obs.	ϕ	T'_A obs.	T'_A calcd.	T'_B obs.	ϕ	T'_A obs.	T'_B calcd.	T'_B obs.	ϕ	T'_A obs.	T'_A calcd.	T'_B obs.	ϕ	T'_A obs.	T'_A calcd.	T'_B obs.	ϕ	T'_A obs.	T'_A calcd.
319.0	170.5	293.1	293.7	284.7	80.4	293.1	293.0	274.9	59.2	273.1	273.4	276.9*	63.3	353.1*	353.1	288.7	100.6	273.1	272.7
323.0*	182.4	298.1*	298.1	291.0	94.2	303.1	303.4	276.1*	61.7	283.2*	283.2	289.1	89.9	373.1	372.8	300.2	120.9	283.1	283.0
327.6	195.1	303.1	302.9	293.8*	100.9	308.1*	308.1	277.3	64.1	293.1	295.1	296.0	106.3	383.1	384.1	311.6*	135.7	293.1*	293.1
332.5	209.9	308.1	308.3	296.7	108.2	313.1	313.0	280.3	70.7	323.1	325.3	304.9	129.8	398.1	398.9	323.0	151.2	303.1	303.2
336.6	223.7	313.1	313.0	299.5	115.6	318.1	317.8	285.2	81.5	372.1	375.8	318.7	168.9	423.1	422.1	334.4	167.1	313.1	313.2
341.0	238.0	318.1	317.9	302.6	123.3	323.1	323.1	287.2	85.4	397.1	396.3					346.0	184.0	323.1	323.4
345.6	253.7	323.1	323.1	305.6	131.5	328.1	328.3	289.8	91.6	427.1	424.7					357.2	201.2	333.1	333.2
350.0	269.5	328.1	328.1	308.5	139.8	333.1	333.4									368.3	219.3	343.1	342.8
354.4	286.5	333.1	333.2	311.6	147.9	338.1	338.8									383.1	243.6	355.7	355.6
																393.1	261.4	364.3	364.2

[8] Bingham, E. C., and J. L. Cadwell, *Z. physik. Chem.*, **83**, 649–50 (1913).
[9] Bingham, E. C., and G. F. White, *ibid.*, **83**, 646 (1913).
[10] Bingham, E. C., and G. F. White, *loc. cit.*, 647.
[11] Koch, S., *Wied. Ann.*, **14**, 1 (1881).
[12] Bramley, A., *J. Chem. Soc.*, **109**, 10 (1915).
[13] Landolt-Börnstein, p. 79 (1912).
[14] *Ibid.*, p. 78 (1912).

determined at 125° C. by Bramley [18] and by Kurnakow, Krotkow and Oksman [19]).

It was thought that the surface tension (γ) and absolute temperature of liquids might also be connected by an equation having the form, $R' = R + c(T'_B - T_B)$, where R' and R are the ratios of the absolute temperatures at which two liquids, A and B, have the same surface tensions. Unfortunately, it has only been possible to test the relation with a few pairs of liquids, owing to lack of experimental surface tension data covering a wide range. With the pairs of liquids studied, the agreement between the

TABLE III.

Liquids.		Fluidity Intervals.	c
A	*B*		
Pyridine [15]	Water [9]	75.6–259.7	0.00200
Acetic acid [16]	Propionic acid [17]	80.4–164.6	−0.000194
Propionic acid [17]	Water [9]	84.5–202.0	0.00218
Aniline [15]	Water [9]	64.7–157.0	0.00174
Quinoline [15]	Water [9]	59.8–182.8	0.00370
Dimethylaniline [15]	Water [9]	49.4–152.0	0.00267
Phenetole [15]	Water [9]	53.8–179.2	0.00184
Ethyl alcohol [16]	Water [9]	91.0–212.5	not constant

observed and calculated temperatures is good, as is illustrated by the data recorded in Table IV. In addition to those given in this table, the following pairs of liquids [21] have been studied: anisol and diethyl chloromaleate, diethyl phenylpropionate and thiophenol, diethylmaleate and toluene, and triphenylphosphine and triphenylstibine. It will be observed that not only does the relation hold for non-associated liquids (*e.g.*, bromtoluene and chlorbenzene), but that it also holds for an associated and a non-associated liquid (*e.g.*, acetic acid and toluene). In view of the good agreement obtained with the few examples studied, it is probable that, when additional surface tension data become available, the expression, $R' = R + c(T'_B - T_B)$, will be found to hold for other pairs of liquids.

[15] Bramley, A., *J. Chem. Soc.*, **109**, 10, 434 (1916).
[16] Bingham, E. C., and G. F. White, *loc. cit.*
[17] Dunstan, A. E., *J. Chem. Soc.*, **107**, 667 (1915).
[18] *Loc. cit.*
[19] *Bl. Acad. Sc. Pét.*, **9**, 45.

It has also been found that the equilibrium pressure (reaction or decomposition pressure) of various chemical reactions in solid-gas, or wholly gaseous, systems can be calculated at different temperatures by means of the expression, $R' = R + c(T'_B - T_B)$. Here R' is the ratio of the absolute temperatures at which two reactions, A and B, have the same equilibrium pressure, and R is the ratio of the absolute temperatures corresponding to another value of the equilibrium pressure, the same for both reactions. A number of types of reactions has been studied and, for pairs

TABLE IV.

Calculation of Absolute Temperatures Corresponding to Given Surface Tensions.

Acetic Acid[20] (A) and Toluene[21] (B); $c = 0.000270$				Bromtoluene[22] (A) and Chlorbenzene[21] c = zero			
T'_B obs.	γ	T'_A obs.	T'_A calcd.	T'_B obs.	γ	T'_A obs.	T'_A calcd.
299.0	26.87	303.1	302.6	287.7	32.992	283.1	282.9
308.6*	25.84	313.1*	313.1	298.1*	31.727	293.1*	293.1
317.9	24.82	323.1	323.3	308.3	30.441	303.1	303.1
327.0	23.80	333.1	333.4	318.3	29.521	313.1	313.0
335.7	22.80	343.1	343.1	328.5	28.079	323.1	323.0
344.8	21.79	353.1	353.2	338.3	26.925	333.1	332.6
353.9	20.81	363.1	363.5				
371.5	18.90	383.1	383.2				

of reactions of the same general type, the agreement between the observed and calculated results has been good in most cases. With pairs of reactions of different types, *e.g.*,

(A) $AgP_2 \rightleftarrows AgP + P$, and
(B) $2CuSO_4 \rightleftarrows 2CuO + 2SO_3$,

the expression does not hold for two out of the three pairs studied. This may, however, be due to inaccurate experimental data rather than to anomalous behavior of the reacting substances. The results obtained with three pairs of reactions are given in Table V, where it will be observed that the calculated absolute temperatures at which reaction A has definite equilibrium pressures agree closely with the temperatures determined by experiment. Other pairs of reactions that have been studied are recorded in Table VI.

[20] Bennett, J. M., *J. Chem. Soc.*, **107**, 351 (1915).
[21] Walden, P., and R. Swinne, *Z. physik. Chem.*, **79**, 708 (1912).
[22] Müller, A. H. R., *ibid.*, **86**, 177 (1914).

TABLE V.

Calculation of Absolute Temperatures Corresponding to Given Equilibrium Pressures.

(A) $2(Cr_2O_3.CuO)+2CuO \rightleftarrows 2(Cr_2O_3.Cu_2O)+O_2$[22] (B) $2BaO_2 \rightleftarrows 2BaO+O_2$[24] c = zero				(A) $Ca(OH)_2 \rightleftarrows CaO+H_2O$[25] (B) $Ba(OH)_2 \rightleftarrows BaO+H_2O$[25] c = -0.000170				(A) $NH_4Br \rightleftarrows NH_3+HBr$[26] (B) $PH_4Cl \rightleftarrows PH_3+HCl$[27] c = zero			
T'_B obs.	Equilibrium Pressure mm.	T'_A obs.	T'_A calcd.	T'_B obs.	Equilibrium Pressure. mm.	T'_A obs.	T'_A calcd.	T'_B obs.	Equilibrium Pressure. mm.	T'_A obs.	T'_A calcd.
982.5*	176	1052*	1052	903	9.2	642	638	194	6	524	524
1020	322	1092	1092	943	17.4	662	660	203*	21	548*	548
1033	410	1106	1106	983	31.5	681	681	209	43	569.5	565
1039	440	1113	1113	1022*	55	701*	701	213	63.5	582	575
1048	512	1121	1121	1062	92	721	721	228	195	616	616
1058	615	1133	1133	1102	149	741	741	233	289	629	629
				1143	234	761	761	236	363	637	637
				1183	355	780	779	241	532	652	651
				1224	526	800	802	244	645	659	659
				1271	760	820	818	247	813	668	667

[22] Wöhler L. and P., *Z. physik. Chem.*, **62**, 448 (1908).
[24] Chatelier, H. le, *Compt. rend.*, **115**, 655 (1892).
[25] Johnston, J., *Z. physik. Chem.*, **62**, 347 (1908).
[27] Johnston, J., *ibid.*, **65**, 38 (1909).

The data given in this paper show that, by means of an expression precisely similar to the Ramsay and Young vapour pressure equation, it is possible to calculate the fluidity (or viscosity) of liquids, the equilibrium pressure of chemical reactions and prob-

TABLE VI.

	Reactions	Pressure Intervals mm.	Constant c.
1.	(A) $2BaO_2 \rightleftarrows 2BaO+O_2$[24]	65–670	zero
	(B) $2PbO_2Ca \rightleftarrows 2PbO_2Ca+O_2$[24]		
2.	(A) $2(Cr_2O_3.CuO)+2CuO \rightleftarrows 2(Cr_2O_3.Cu_2O)+O_2$[25]	176–795	zero
	(B) $2PbO_2Ca \rightleftarrows 2PbO_2Ca+O_2$[24]		
3.	(A) $2CuBr_2 \rightleftarrows 2CuBr+Br_2$[25]	22–597	not constant
	(B) $2BaO_2 \rightleftarrows 2BaO+O_2$[24]		
4.	(A) $Sr(OH)_2 \rightleftarrows SrO+H_2O$[25]	9.2–760	0.000093
	(B) $Ba(OH)_2 \rightleftarrows BaO+H_2O$[25]		
5.	(A) $Mg(OH)_2 \rightleftarrows MgO+H_2O$[25]	9.2–760	−0.000062
	(B) $Ba(OH)_2 \rightleftarrows BaO+H_2O$[25]		
6.	(A) $Fe_2(SO_4)_3 \rightleftarrows Fe_2O_3+3SO_3$ $[SO_2.O_2]$[29]	33–560	zero
	(B) $Al_2(SO_4)_3 \rightleftarrows Al_2O_3+3SO_3$ $[SO_2.O_2]$[29]		
7.	(A) $NH_4Br \rightleftarrows NH_3+HBr$[30]	6–532	zero
	(B) $NH_4I \rightleftarrows NH_3+HI$[30]		
8.	(A) $ZnSO_4 \rightleftarrows ZnO+SO_2$ $[SO_2.O_2]$[29]	24–189	−0.00099
	(B) $2AuCl \rightleftarrows 2Au+Cl_2$[31]		
9.	(A) $AgP_2 \rightleftarrows AgP+P$[32]	59–214	not constant
	(B) $2CuSO_4 \rightleftarrows 2CuO+2SO_3$ $[SO_2.O_2]$[29]		

ably the surface tension of liquids. The agreement between the calculated and observed data presented in the preceding tables indicates that values obtained by this method represent a close approximation of the truth. The utility of the expression $R' = R + c(T'_B - T_B)$ is evident, for if values of one of the physical constants are known over a wide range of temperatures for one substance (B), and if the value of the constant is determined at a few temperatures for a second substance (A), then its value for the

[24] Tammann, G., "Krystallisieren und Schmelzen," 1903, p. 289.
[25] Jackson, C. G., *J. Chem. Soc.*, **99**, 1066 (1911).
[29] Wöhler, L., W. Püddemann and P. Wöhler, Ber., **41**, 703 (1908).
[30] Johnston, J., *Z. physik. Chem.*, **65**, 38 (1909).
[31] Meyer, F., *Compt. rend.*, **133**, 817 (1901).
[32] Granger, *Ann. chim. phys.*, (7), **14**, 5 (1901).

latter can be calculated over a range corresponding to the range of the known values for the first substance (B). To do this, the values of the ratio $T'_A/T'_B = R$ are plotted against the absolute temperatures T'_B and a straight line drawn through the points. By multiplying a particular temperature, T'_B, by the corresponding temperature ratio (read off from the curve), the absolute temperature, T'_A, is obtained at which the value of the physical constant of the substance A is equal to that of the substance B at the absolute temperature T'_B. If the line does not run through the points (*i.e.*, if c is not constant), it is probable that the experimental data are not accurate, or that one of the substances behaves abnormally.

SWARTHMORE, PA.,
March 16, 1922.

The Atomic Process in Ferromagnetic Induction. SIR J. A. EWING. (*Proc. Royal Society*, A 706.)—In 1890 the author brought to attention a model which embodied his conception of ferromagnetic induction. In one plane it consisted of a group of similar, pivoted magnets. When no external magnetic force acts, these form configurations under the influence of their mutual attractions and repulsions, and when an external force is brought into action, as by the insertion of the group of magnets into a magnetizing coil, it is their mutual forces which must be overcome before the magnets can turn into the direction of the applied magnetizing force. Such a model simulated many of the phenomena manifested when a piece of iron is magnetized and, indeed, with striking completeness. Professor Ewing, however, finds that there is a great discrepancy between the magnetizing force really needed to make iron pass from the reversible to the irreversible stage and that which would be needed to accomplish the same change were the elementary magnets subject alone to their mutual attractions. He finds it necessary to place near each pivoted magnet two pairs of permanent magnets. Each pair has poles arranged thus, N S–S N and the axes of both magnets lie in the same line, but the directions of these lines are different for the two pairs. The pivoted magnet can turn within the magnets forming a pair without touching either. There is assumed to be a slight dissymmetry in the strength or in the position of the two magnets in the same line.

When it comes to seeking in the actual atom something akin to this highly artificial arrangement, it is suggested that a pair of electrons on opposite sides of the nucleus of the atom may correspond to the pivoted magnet, while groups of electrons farther away from the centre may play the rôle of the fixed magnets. The new type of model is far removed from the simplicity of its predecessors of 1890,

which did so much to clarify magnetic conceptions when hysteresis was a new thing. Magnetism is, however, in itself no simple matter and investigation seems to increase the number of problems awaiting solution rather than to furnish a clue to already existing difficulties. It would therefore appear that a complex model is needed to represent complex effects and surely no one is better fitted to make fruitful suggestions in this field than one who has already succeeded eminently.

G. F. S.

A New Hydrogen Process.—Much attention has been paid of late years to methods of obtaining hydrogen, principally on account of its use in the treatment of fats and for the inflating of dirigibles. It has also some applications as a reducing agent and as fuel. The use of ferrosilicon in solution of sodium hydroxide is one of more recent methods, which is described in a paper by E. R. Weaver, presented at the forty-first meeting of the American Electrochemical Society in April last. Although more expensive than other processes, it is suitable for military use on account of the small cost of the plant and the simplicity of operation. The reaction is between the silicon and the alkali, and the principal equation is

$$2NaHO + Si + H_2O = Na_2SiO_3 + 2H_2.$$

This represents the change at the beginning of the operation, but secondary reactions occur by the hydrolysis of the sodium silicate. The temperature must be controlled during the process. The facts that none of the materials used are combustible, that they do not give off any gas until mixed, and are easily transportable, are especially advantageous in naval operations. The gas produced is of high purity, traces of phosphine and acetylene being the principal impurities, although hydrocarbons and even arsine may occur. The disposal of the exhausted liquor is, however, a serious matter, as it is too alkaline to be thrown into a running stream.

H. L.

Latent Heat of Fusion. MRS. K. STRATTON and J. R. PARTINGTON. (*Phil. Mag.*, March, 1922.)—The method seems to be new. A known mass of solid is maintained at its melting point by being surrounded by a mixture of the solid and liquid phases of the same substance. Heat is then imparted to the mass by sending an electric current through a resistance coil. This is continued until all the solid has just turned into liquid. Of course adequate stirring is provided. From the energy added and the mass of the solid it is easy to calculate the latent heat of fusion. In the case of benzophenone three separate experiments gave these values for the specific heat, 21.64, 21.79 and 21.58 calories per gram. An attempt was made to get the latent heat of rhombic sulphur, but it failed owing to the transformation into the monoclinic variety that occurred during the course of the experiment.

G. F. S.

COMMUTATION IN CONTINUOUS CURRENT DYNAMOS.*

BY

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THIS article is an attempt to explain in the simplest possible manner the theory of commutation by emphasizing the physical aspects of the problem and avoiding mathematical expressions and formulas of which the meaning and derivation cannot be easily understood. With this end in view, considerations of a mechanical nature, such as vibration, uneven or oily commutator surface, insufficient or excessive brush pressure, etc., will not be considered. Ideal or "straight-line" commutation will be assumed, and the conditions necessary to produce this—generally desirable—result investigated, in order that a multitude of more or less arbitrary assumptions may not obscure the problem in its early stages. By working from the simplest possible case to the more complex it is thought that the object in view—a physical conception of commutation phenomena leading to practical ends—will best be served, and influencing factors of relatively small practical importance will be either disregarded or but briefly referred to.

The theoretical investigation of commutation phenomena is admittedly difficult, partly because it is impossible to take account of the many causes which may lead to sparking at the brushes, but also because there can be no general solution of the problem applicable to the many different designs and windings of continuous current armatures. It is for these reasons that the writer does not hesitate to adopt the method proposed by Mr. Lamme,[1] who was the first to put the theory of commutation in a new light, thus clearing away much of the vagueness, if not inaccuracy, which accompanied the earlier methods of studying commutation phenomena.

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* Communicated by the Author.

[1] B. G. Lamme, "A Theory of Commutation and its Application to Interpole Machines," *Trans. A.I.E.E.*, vol. xxx, pp. 2359–2404.

A continuous current dynamo is provided with a commutator in order that unidirectional currents may be drawn from armature windings in which the current actually alternates in direction as the conductors pass successively under poles of opposite kind. As each coil in turn passes through the zone of commutation, it is short-circuited by the brush, and during the short lapse of time between the closing and opening of this short-circuit the current in the coil must change from a steady value of $+I_c$ to a steady value of $-I_c$.

Let W = surface width of brush (brush arc) in centimetres.
M = thickness of insulating mica in centimetres.
V_c = surface velocity of commutator in centimetres per second.

The time of commutation in seconds may then be written,

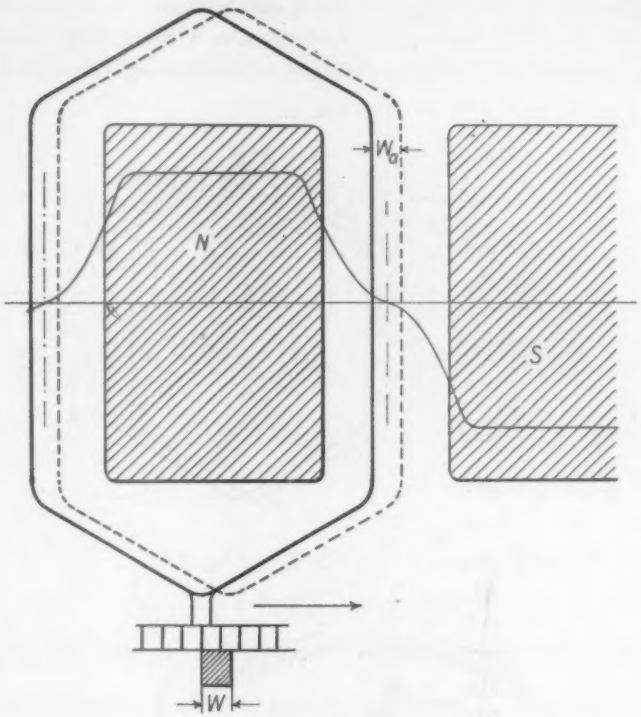
$$t_c = \frac{W - M}{V_c}$$

Since M is usually small with reference to W, it is generally permissible to express the time of commutation as $t_c = \frac{W}{V_c}$; that is to say, the time taken by any point on the commutator surface to pass under the brush is approximately the same as the duration of the short-circuit. It is during this time, t_c, that the current in the commutated coil must pass through zero value in changing from the full armature current of $+I_c$ to the full armature current of $-I_c$. If R is the resistance of the short-circuited coil, and if any possible disturbing effect of brush contact resistance be neglected, it is evident that the e.m.f. in the coil should be $e = I_c \times R$ at the commencement of commutation. At the instant of time when the current is changing its direction (*i.e.*, when no current is flowing in the coil) the e.m.f. is $e = O \times R = O$. At the end of the time t_c, when the coil is just about to be thrown in series with the other coils of the armature winding carrying a current $-I_c$ amp., the e.m.f. in the coil should be $e = -I_c R$. It is when the e.m.f. in the coil has some value other than this ideal value that sparking is liable to occur.

Theory of Commutation.—Consider a closed coil of wire of T_c turns moving in a magnetic field. At the instant of time $t = O$ the total flux of induction passing through the coil is $+\phi_o$ maxwells, and at the instant of time $t = t_c$ sec. the total flux

through the coil is $+\phi_t$ maxwells. Then, on the assumption that the flux links equally with every turn in the coil, the average

FIG. 1.



value of the e.m.f. developed in the coil during the interval of time t_c is

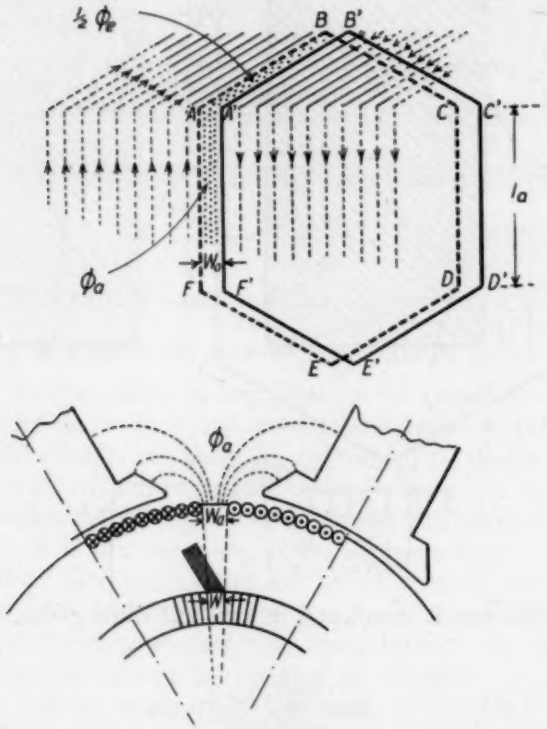
$$E_m = \frac{(\phi_t - \phi_o)\, T_c}{t_c \times 10^8} \text{ volts.} \qquad (1)$$

If R is the ohmic resistance of the coil and e is any instantaneous value of the e.m.f. produced by the cutting of the actual magnetic field in the neighborhood of the coil, the instantaneous value of the current in the coil is $i = \frac{e}{R}$ because e is the only e.m.f. in the circuit tending to set up a current. The usual conception of a distinct flux due to the current i producing a certain flux linkage known as the self-inductance of the circuit

is avoided; but its equivalent has not been overlooked seeing that the magneto-motive force due to the current in the coil is a factor in the production of the flux actually linked with this current at the instant of time considered.

Following the lead of Mr. Lamme, the wires in the coil undergoing commutation will be thought of as cutting through a total

FIG. 2.

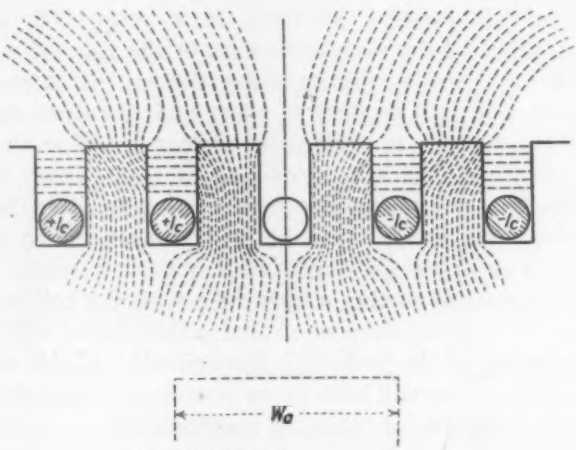


flux of induction, expressed in magnetic lines or maxwells, this flux being the resultant of the magnetizing forces of field coils and armature windings combined.

If a full-pitch armature coil is short-circuited by the brush while half-way between the *N* and *S* pole tips—that is to say, while the coil sides are in the geometric neutral zone—the magnetic fluxes due to the main poles are cancelled and their effect need not be considered. It is then only the flux set up by the armature winding itself (considered separately from the field

fluxes) which is effective in producing an e.m.f. in the coil undergoing commutation. The fact that the flux from the field poles which passes through the short-circuited coil is the same at the end as at the beginning of commutation—when this occurs in the "geometric" neutral zone—is clearly shown in Fig. 1. Here W_a is the distance of travel of the armature conductors

FIG. 3.



Flux distribution in commutation zone.

during the time of short circuit: it may be defined as the brush width referred to the armature periphery. Thus $W_a = W\frac{D}{D_c}$ where W is the brush width (or brush arc), D is the diameter of the armature and D_c is the diameter of the commutator. The curve shows the distribution of flux density over the armature surface, being positive, or (say) from the N pole when measured above the datum line, and negative, or of opposite polarity, when measured below the datum line. It is obvious that with all poles of the same size and shape, excited with the same number of ampere turns, the flux density will be zero on a line exactly half-way between the poles; and since the resultant flux through a coil of width equal to pole pitch is exactly the same for both positions of the coil shown in the figure, it follows that the *mean* voltage developed in the coil while moving through the distance W_a is zero. If this distribution of flux in the com-

mutating zone were not disturbed by the currents in the armature conductors, commutation would offer no serious difficulties even under heavy loads.

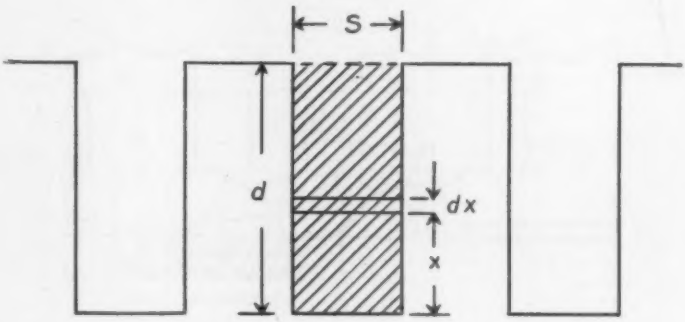
Magnetic Flux Due to Armature Currents.—The brushes are still supposed to be in such a position that the coil sides are at a point midway between poles when the mica which separates the two ends of the coil is under the centre of the brush. When considering the changes in flux density brought about by the currents in the armature windings, it is permissible and convenient to think of the magnetic fluxes in the zone of commutation, which are due to the armature current, as distinct from other fluxes, and also as *fixed in space* and therefore *cut* by the armature conductors as they move across it. For those who may experience difficulty in conceiving of a *stationary* flux[2] due to currents in *moving* armature conductors which are themselves cutting this same flux, it may be more convenient to revert to formula (1) and consider merely what is the difference $(\phi_t - \phi_o)$ between the amount of flux which links with the coil at the end of the period of short circuit and that which linked with the coil at the beginning of the period of short circuit. If this difference is zero, commutation will have taken place in a neutral field which is the ideal condition for sparkless commutation.

In Fig. 2, the dots between the two extreme positions of the coil undergoing commutation indicate flux lines, all of the same polarity, which are due to the armature currents only. The fact that there will be some flux, ϕ_a, due to the ampere turns on the armature in the zone of width W_a and length l_a (*i.e.*, the gross length of the armature core) is generally understood, but that the same is true of the end connections is not always so clearly realized. It will, however, be evident from an inspection of Fig. 2 that flux of the same kind as that in the space $FAA'F'$ will be found in the space $ABB'A'$, although it may be of lower density. This flux is due to the conductors running parallel to $A\ B$ carrying currents which are opposite in direction on the two sides of the space considered. What has been said of the space $ABB'A'$

[2] It is true that there are certain flux oscillations and pulsations which depend upon the slot pitch, the number of coil sides in a slot, and the relation of brush width to width of commutator bar, but these produce only minor variations in the e.m.f. induced in the coil during commutation, and since these can usually be taken care of by the surface resistance of carbon brushes, their existence will be ignored for the present.

applies equally to the flux cut by portion $F\ E$ of the end connections, and if the coil-side $EFAB$ cuts through a positive flux $(\phi_a + \phi_e)$ it will readily be seen that coil-side $BCDE$ cuts through an equal amount of negative flux, so that the difference in the amount of flux enclosed by the coil at the end and beginning of commutation (the quantity $\phi_t - \phi_o$ of formula (1)) is 2 $(\phi_a + \phi_e)$. This difference of flux will always occur unless the brushes are shifted so as to bring the short-circuited conductors into a field

FIG. 4.

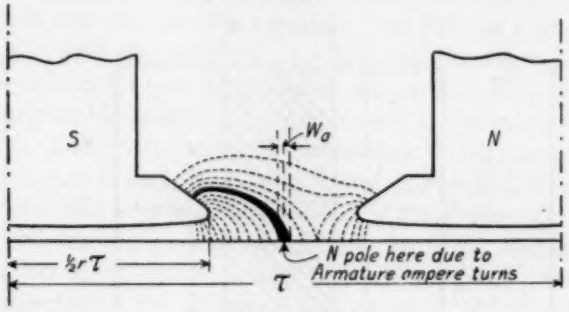


from the main poles sufficient to annul it, or unless commutating poles are provided.

Whether in a dynamo or motor, the flux due to the armature currents is always such as to set up e.m.f.s in the short-circuited coil tending to oppose reversal of the current. It is for this reason that the important element in the problem of commutation, from whatever point of view it is approached, is, and always will be, the correct determination of the field in which the short-circuited coil is moving, whether this conception of the magnetic condition is buried in the symbols L and M, and referred to as inductance, expressed in henrys, or considered merely as any other magnetic field; which is the method followed in this article. Although the end flux (the quantity ϕ_e) is not easily calculated, it is rarely of negligible amount and should not be left out of consideration. It is true that we do not concern ourselves with the end fluxes when calculating the useful voltage developed in the active coils; but, apart from the fact that in this connection the amount of the end flux is relatively small, it is not difficult to see that the e.m.f.s generated in the end connections as they cut

through the end fluxes due to the armature currents balance or counteract each other and have no effect upon the terminal voltage. The conception of the end connections cutting through the flux due to the armature as a whole, as indicated in Fig. 2, seems more natural, and is more helpful to the understanding of commutation phenomena, than what might be termed the academic method, in which more or less reasonable assumptions are made in respect to self and mutual inductances; but it is not

FIG. 5.



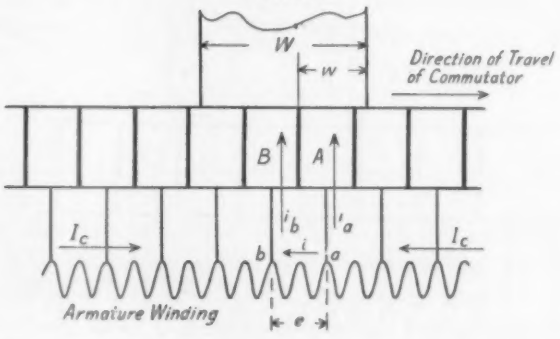
suggested that one method is necessarily superior to the other so far as practical results are concerned. While moving from the position at the commencement of commutation where the current is $+I_c$ to the position at the end of commutation where the current is $-I_c$, the short-circuited coil has cut through the flux of self and mutual induction—through the whole of it, not merely through certain components of the total flux in the particular region considered. This is well expressed by Mr. Menges when he says [3]: " . . . Self-induction is in no way distinguishable from other coexistent electromagnetic induction. Therefore, when the real magnetic flux resulting from all causes, and its changes relative to a given circuit, are taken into account, the self-induction is already included, and it would be erroneous to add an e.m.f. of self-induction."

Effect of Slot Flux.—Practically all modern dynamos and motors have slotted armatures and since the effect of the slot leakage flux on commutation has not yet been discussed, it will be necessary to consider it before proceeding farther. In Fig. 3

[3] C. L. R. E. Menges in the *Electrician*, Feb. 28, 1913.

an attempt has been made to represent, by the usual convention of magnetic lines, the flux due to the armature current alone, which enters or leaves the armature periphery in the interpolar space when the field magnets are not excited. The position chosen for the brushes is the geometric neutral (*i.e.*, the point midway between two poles) and the magnetic lines leaving the teeth will cross the air spaces between armature surface and field poles and so close the magnetic circuit. The brush is supposed to cover

FIG. 6.



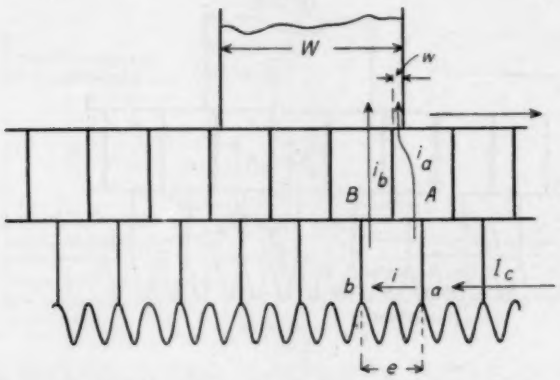
an angle equal to twice the slot pitch. The current in the conductor just before the short-circuit occurs is $+I_c$, the current in the conductor occupying the slot corresponding to a position under the centre of the brush is zero, and the current in the conductor just passing out from the short circuit is $-I_c$. The armature is supposed to be rotating, and it will be seen that the conductors in which the current is being commutated are cutting the flux set up by the armature as a whole. It is important to note that the flux cut by a conductor while travelling between the two extreme positions during which the short circuit obtains is not limited to the flux passing into the air gaps from the tops of the teeth included between these positions of the conductor, but includes also the flux due to the currents in the short-circuited conductors, which crosses the slot above the conductor [4] and leaves the armature surface by teeth which are not included between the two extreme positions of the short-circuited coil. This picture of the

[4] For the sake of simplicity, a single conductor is shown at the bottom of each slot and the whole of the slot flux is supposed to link with it. The calculation of the "equivalent" slot flux will follow.

conductor cutting the field set up by the armature currents is especially useful when calculations are made, as will frequently be found useful, by considering the separate component fluxes due to distinct causes, all combining to produce the actual or resultant flux. It is not difficult to see that the flux shown in Fig. 3 is never such as to generate an e.m.f. tending to reverse the current in the short-circuited conductor.

Calculation of Slot Flux.—Instead of all the slot flux passing

FIG. 7.



above the conductors to which it owes its existence, as shown in Fig. 3, much of this flux will actually pass through the copper in the slot. In order to simplify the calculations and yet obtain formulas of sufficient accuracy for the purpose of the designer, the whole of the slot—including the space usually occupied by the wedge—will be supposed occupied by the coils, the slot dimensions (in centimetres) being as indicated in Fig. 4. It will be convenient to assume the same number of commutator bars as there are slots, and each slot to be filled with $2T$ conductors, each carrying I_c amp. (This follows from the assumption of a full-pitch winding.) Thus no account will be taken of the fact that a small space occurs between upper and lower coils where the slot flux will not pass through the material of the conductors. The lines of the slot flux will be supposed to take the shortest path from tooth to tooth; the small amount of flux that may follow a curved path from corner to corner of tooth at the top of the slot will be neglected. Refinements of this nature may be introduced, if desired, when solving the problem for a concrete case.

If the usual assumption is made that the reluctance of the iron in the path of the magnetic lines is negligible in comparison with the slot reluctance, the small portion of slot flux in the space dx (Fig. 4), considered one cm. long axially (*i.e.*, in a direction perpendicular to the plane of the paper) is

$$d\phi = \text{m.m.f.} \times dP$$

where dP is the permeance of the air path. Thus

$$d\phi = 0.4\pi (2\,T\,I_c)\frac{x}{d} \times \frac{dx}{s}$$

and the total slot flux *per centimetre of axial length* in the zone of commutation—being *twice* the flux per slot (see Fig. 3)—is

$$\phi_s = \frac{0.8\pi (2\,T\,I_c)}{ds} \int_0^d x\,dx$$

$$= \frac{0.8\pi d\,TI_c}{s} \qquad (2)$$

Since this flux is not cut equally by all the conductors in the slot the volts generated in the short-circuited coil by the cutting of this leakage flux will depend upon what may be termed the *equivalent* slot flux. This may be defined as the amount of flux which, if cut by all the conductors in the slot, would generate in the coil the same e.m.f. as results from the cutting of the actual slot flux. Thus, the element of flux in the space of depth dx (Fig. 4) links only with $2\,T\left(\frac{x}{d}\right)$ conductors,[5] and the equivalent amount of flux which, if cut by all the conductors in the slot, would generate the same e.m.f. is therefore

$$d\phi_{(\text{equiv.})} = d\phi \times \frac{x}{d}$$

whence

$$\phi_{(\text{equiv.})} = \frac{0\cdot 8\,\pi\,T\,I_c}{d^2\,s} \int_0^d x^2\,dx$$

$$= \frac{0\cdot 8\,\pi\,d}{3\,s}\,T\,I_c \text{ maxwells per slot.}$$

The total slot flux cut by each coil side during commutation,

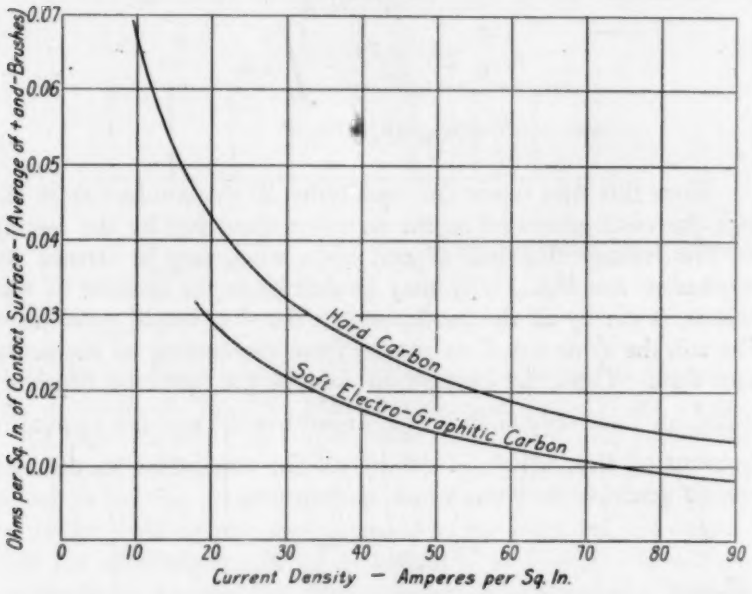
[5] These calculations are based on the assumption of a large number of turns in the armature coils. When the winding consists of only one or two conductors of large cross-section in each slot, the current distribution will not be quite uniform as here assumed.

being twice the flux per slot, as previously explained, the final expression for the equivalent slot flux per centimetre axial length of the armature when the path of the flux lines is up through the root of the tooth and outward across the slot through the copper of the conductors, is

$$\phi_{es} = \frac{1 \cdot 6 \pi d}{3 s} T I_c \quad \text{maxwells.} \qquad (3)$$

Formulas for Calculating Armature Flux and End Flux in the Zone of Commutation.—It is not possible to develop an accurate

FIG. 8.



general formula for the amount of the "armature flux" cut by the short-circuited coils. By "armature flux" is meant the flux due to the ampere turns on the armature which passes between the field poles and the tops of the armature teeth comprised in the zone of commutation (the space W_a of Figs. 1 and 2). The path taken by the flux lines is not easily predetermined, and it will depend not only upon the slope and distance apart of the pole shoes, but also upon the position of the brushes. A formula of sufficient accuracy for practical purposes may be developed as follows:

The flux entering the zone of commutation of width W_a exactly half-way between the two pole tips is due to the armature m.m.f. only, because the m.m.f.s of the two field poles neutralize at this point. When the field poles are excited, the flux path will be between the armature core and the field pole of opposite kind, generally as indicated in Fig. 5. The length of this flux path, whether it terminates at the pole shoe or higher up on the side of the pole core, depends mainly upon the distance between the pole tips, or $(1-r)\tau$ where τ = pole pitch and r is the ratio $\frac{\text{pole arc}}{\text{pole pitch}}$. It can be shown that, when r has a value between 0.6 and 0.8 and the number of poles is not less than six, the reluctance of this flux path per square centimetre of armature surface is approximately $\frac{\pi}{4}\tau\,(1-r)$ where τ is in centimetres. The density of the flux entering the armature core at a point half-way between the poles when the position of the brushes is such that commutation takes place with the coils in the geometric neutral zone, is

$$B_a = 0.4\,\pi\left(\frac{ZI_c}{2p}\right)\frac{4}{\pi\,\tau\,(1-r)}$$

where Z = the total number of armature conductors carrying a current I_c, and p = number of poles; the quantity in brackets being the armature ampere turns per pole.

If D is the diameter of the armature core in inches, we have $\tau = \frac{\pi D \times 2.54}{p}$ which leads to the simplified expression

$$B_a = \frac{ZI_c}{10(1-r)D} \text{ gausses.} \qquad (4)$$

A more correct formula, which takes account of the increased length of flux path with the curvature as the number of poles is reduced, is

$$B_a = \frac{ZI_c}{7\left(1+\frac{2}{p}\right)(1-r)\,D} \text{ gausses.} \qquad (5)$$

Formulas for calculating the flux cut by the end connections of the armature winding are not applicable to all sizes of machines and types of winding, and they should be considered as approxi-

mations only unless they contain empirical constants which have proved to be reasonably accurate for the particular type of armature to which they apply. By making certain assumptions the writer has developed elsewhere[6] a formula which he has found to be sufficiently accurate for the purposes of the designer. This gives the flux cut by the two ends of one coil side (*i.e.*, half the end connections of a complete armature coil) as

$$\Phi_e = 0.4\sqrt{2}\, k\, W_a\, n\, TI_c \left[\log_e \frac{2\tau}{W_a} - 1\right]$$

where n = number of slots in the space of one pole pitch, and k has a value between 1.5 and 3, the highest value being used when the overhanging ends of the armature coils are secured against iron supporting rings by steel wire bands. The dimensions τ and W_a must be in centimetres.

A close approximation to the quantity in brackets is $0.6\sqrt{\frac{\tau}{W_a}}$ and by assuming the total axial projection of the armature coils outside of the slots to be 0.44τ, the formula becomes

$$\Phi_e = 0.3\, k\, W_a\, n\, TI_c \sqrt{\frac{\tau}{W_a}} \quad \text{maxwells.} \qquad (6)$$

With the aid of these formulas it is possible to determine approximately the average total e.m.f. developed in the coil during the time of commutation by the cutting of flux lines due to currents in the armature windings. It should be remembered that this e.m.f. is always such as to oppose the reversal of the current in the short-circuited coil.

Calculation of Short-circuit E.M.F.—Numerical Example.—In order to illustrate the manner in which the approximate value of the e.m.f. induced in the coil during short-circuit may be calculated, the dimensions and other particulars of the machine will be assumed as follows:

R. P. M. = 500
Number of poles $p = 6$
Diameter of armature core $D = 30$ in.
Ratio $\frac{\text{Pole arc}}{\text{Pole pitch}} = r = 0.625$
Length of armature core $l_a = 11$ in. = 28 cm.
Total number of slots = 120

[6] "Principles of Electrical Design," second edition, p. 155.

Number of slots per pole $n = 20$
Slot Pitch $\lambda = 0.785$ in.
Slot width $s = 0.39$ in.
Slot depth $d = 1.5$ in.
Style of winding: Full pitch, multiple.
Current per armature path $I_a = 76$ amp.
Number of conductors per slot $= 2T = 8$
Total number of armature conductors $Z = 120 \times 8 = 960$
Number of commutator bars $= 120$ (There are only two coil sides per slot, giving four turns between adjacent commutator bars.)
Diameter of commutator $= 20$ in.

Pitch of commutator bars $= \frac{\pi \times 20}{120} = 0.524$ in.
Number of bars covered by brush $= 1\frac{3}{4}$
Brush arc $W = 0.524 \times 1.75 = 0.916$ in.

Brush arc referred to armature periphery,

$$W_a = \frac{0.916 \times 30}{20} = 1.375 \text{ in.} = 3.5 \text{ cm.}$$

With the brushes exactly half-way between pole tips, and a total travel of the coil of $W_a = 1.375$ inches during the time of short-circuit, the total flux cut by each coil side, including one-half of the entire length of the end connections per coil, may be calculated by making use of the formulas for the three separate components of this flux.

By formula (3), the equivalent slot flux is

$$l_a \Phi_{es} = 28 \frac{1.6\pi \times 1.5}{3 \times 0.39} \times 4 \times 76 = 55{,}000 \text{ maxwells.}$$

By formula (5), the armature flux which leaves the teeth in the zone of commutation of width W_a and length l_a is

$$W_a l_a B_a = 3.5 \times 28 \times \frac{960 \times 76}{7(1+\frac{2}{6})(1-0.625)30} = 68{,}000 \text{ maxwells.}$$

The end flux is calculated by formula (6). We shall assume the factor k to have the value 2, and the pole pitch is

$\tau = \frac{\pi \times 30 \times 2.54}{6} = 40$ cm. Then

$$\Phi_e = 0.3 \times 2 \times 3.5 \times 20 \times 4 \times 76 \sqrt{\frac{40}{3.5}} = 43{,}200 \text{ maxwells.}$$

The sum of these three components gives a total of 166,200 maxwells cut by one side of the coil of four turns in the time

$$t_c = \frac{W_a}{\text{peripheral velocity of armature}} = \frac{1.375 \times 60}{\pi \times 30 \times 500} = \frac{1}{572} \text{ sec.}$$

The average value of the e.m.f. generated in the short-circuited coil is therefore,

$$\frac{\text{Twice total calculated flux} \times \text{No. of turns in coil}}{10^8 \times \text{time of cutting (in seconds)}}$$

$$= 2 \times 166200 \times 4 \times 572 \times 10^{-8} = 7 \cdot 6 \text{ volts.}$$

This voltage, which opposes the reversal of current, may be annulled by providing commutating poles or by shifting the brushes so that the resultant of all fluxes cut by the coil during commutation shall be zero.

The surface resistance offered by carbon brushes will correct appreciable departures from the ideal conditions, so that an exact neutralization of all armature fluxes tending to oppose reversal of current is not necessary in modern continuous current machinery.

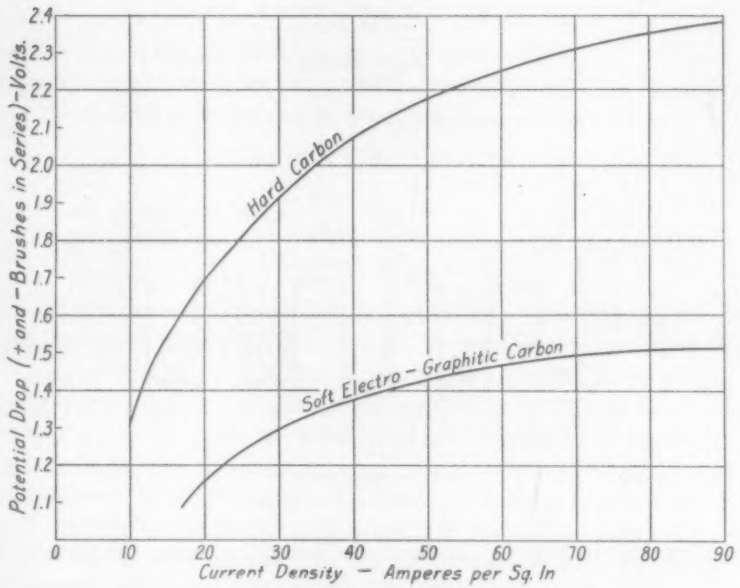
Brush Contact Resistance as an Aid to Commutation.—When copper brushes are used destructive sparking is likely to occur even when a comparatively small e.m.f. is induced in the short-circuited coil. The contact resistance of carbon brushes assists the reversal of current and thus allows of sparkless commutation, even with an appreciable departure from the ideal condition of zero value for the average short-circuit e.m.f. At the beginning and end of the commutation period the field in which the coil moves should be such as to produce an e.m.f. in the short-circuited coil of the value $e = I_c R$, where I_c is the current per path of the armature circuit and R is the resistance of the short-circuited coil. On the assumption of a uniform current density over the surface of the brush, the brush contact resistance need not be taken into account, as will be clear from the following considerations. Fig. 6 shows a brush of width W covering several segments of the commutator. The total current entering the brush is $2I_c$, and since the density is constant over the surface of the contact, the current entering the brush over any surface of width S is $2I_c \frac{S}{W}$. To calculate the volts e that must be developed in the coil of resistance R when the distance yet to be travelled before the end of commu-

tation is w, consider the sum of the potential differences in the local circuit $AabB$ which is closed through the material of the brush. This leads to the equation

$$e = iR - i_a R_a + i_b R_b \qquad (7)$$

where R_a and R_b are contact resistances depending upon the areas

FIG. 9.



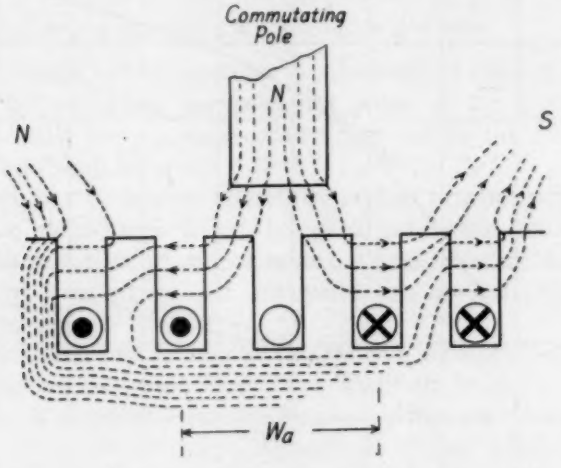
of the surfaces through which the current enters the brush. Under the conditions shown in Fig. 6, the two contact surfaces are equal, and the currents i_a and i_b are therefore also equal. It follows that the voltage drops $i_a R_a$ and $i_b R_b$ are equal and cancel out from the equation (7). The same may be shown to be true when the distance w yet to be travelled by the commutator is less than the width of the brush segment, and it therefore follows that, with " straight-line " commutation and the *uniform current density over the brush contact surface* which necessarily results from the " straight-line " law of current reversal, the only e.m.f. to be developed in the short-circuited coil is $e = iR$.

Thus, although the *average* short-circuit e.m.f. during the time of commutation should be zero, the ideal voltage in the coil

at the beginning and end of commutation is plus or minus I_cR irrespective of the material or contact resistance of the brush.

We shall now consider how, and to what extent, a departure from the ideal conditions is permissible when carbon brushes are used. In Fig. 7 the commutator is shown as having moved forward under the brush, leaving the distance w yet to be travelled before the segment A passes from under the brush considerably

FIG. 10.



smaller than in Fig. 6. If the current density under the portion w of the brush be allowed to exceed the density under the remaining portion of width $(W-w)$, the opposition to the passage of the current component i_a will increase and the greater potential drop at this point will assist in bringing the current i in the coil more nearly up to the value I_c, which it must reach at the instant the mica passes from under the brush. It is true that, under steady conditions, the contact resistance of carbon brushes decreases greatly with increase of current density, as indicated by the curves of Fig. 8, and it might be argued that, since the potential drop (see Fig. 9) is not very much greater with a density of 100 amperes per square inch than with a normal density of 40 amperes per square inch, the corrective effect of the contact resistance of carbon brushes must be very small. However, when the current density is rapidly fluctuating, even between wide limits, it

has been shown[7] that the specific contact resistance will remain approximately constant at all points under the brush.

Let R_c stand for the specific contact resistance of the carbon brush when the average current density per square inch of surface is Δ; and let Δ_w be the greater current density in the space of width w (see Fig. 7) resulting from an induced voltage e in the short-circuited coil which opposes reversal of the current. Then

$$e = i R - \Delta_w R_c + \Delta R_c.$$

but as the space w becomes smaller and smaller, the current i_a decreases and i becomes more and more nearly equal to I_c, so that, for the condition at the end of commutation, we may write

$$e = I_c R - R_c (\Delta_w - \Delta).$$

Let k stand for the ratio $\frac{\Delta_w}{\Delta}$, then

$$e = I_c R - R_c \Delta (k-1)$$

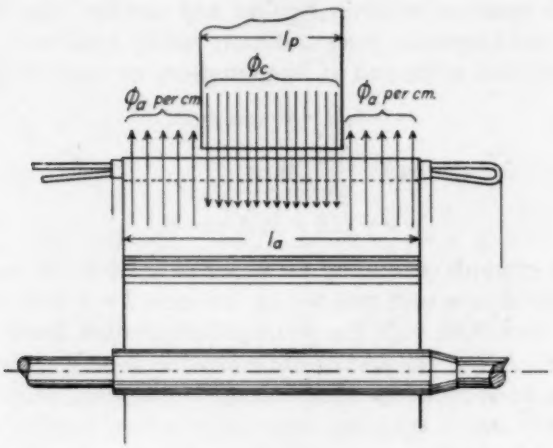
As an example, assuming an upper safe limit for Δ_w of 120 amperes per square inch and $\Delta = 40$, we have $k = 3$, and since $R_c\Delta$ is always about one volt, the permissible variation from the ideal value of $e = I_cR$ at the end of commutation is seen to be 2 volts. If this variation from the ideal voltage is exceeded, there is likely to be trouble due to sparking, even when carbon brushes are used. In practice the high contact resistance of carbon brushes will not only permit slight departures from the correct average value of the reversing field provided by commutating poles (or by the main poles when sparkless commutation is obtained by moving the brushes), but it will also take care of the pulsations and fluctuations in the short-circuit e.m.f., and so prevent great irregularities in the local current passing through the coil *via* the brush contact surface.

The permissible amount of variation from the ideal voltage in the short-circuited coil can be accurately determined only by actual test, because it will depend upon the particular grade of carbon used for the brushes, and other factors which cannot very well be taken account of in the calculations. There are very few designs of dynamos in which the carbon brush alone will bring about satisfactory commutation without the aid of a reversing field to counteract the flux set up in the zone of commutation by

[7] Hawkins and Wallis "The Dynamo," Fifth edition, p. 607.

the currents in the armature windings. Thus, in a previous example the average short-circuit e.m.f. was found to be $e = 7.6$ volts which greatly exceeds what could be taken care of by the contact resistance of carbon brushes. The resistance of the four turns of armature winding which form the short-circuited coil in the example referred to would probably not exceed 0.01 ohm, so that with the current $I_c = 76$ amperes, the ideal value for the

FIG. 11.

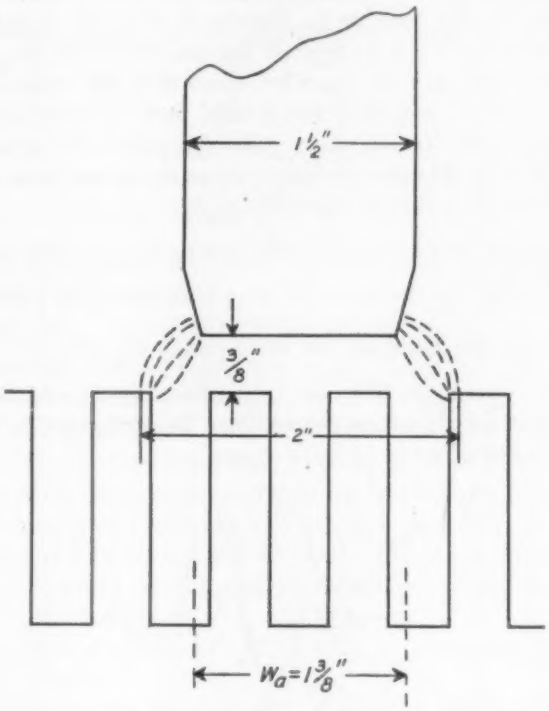


voltage in the coil at the end of commutation would be $e = 76 \times 0.01 = 0.76$ volt, and the difference $7.6 - 0.76 = 6.84$ volts could not be taken care of by the brush contact resistance. Assuming that a variation of 1.5 volts from the ideal value of the short-circuit e.m.f. is permissible, the machine would probably work sparklessly with carbon brushes commutating in the geometric neutral zone with a load of $\frac{1.5}{6.84}$ or (say) 22 per cent. of full load, if the pulsations in the local short-circuit current could be neglected. With a load exceeding 22 per cent. of rated full load it would, however, be necessary to resort to brush shift or provide commutating poles in order to obtain a reversing field and so counteract the effects of the various fluxes in the commutating zone which are due to the armature currents.

Commutating Poles.—These are auxiliary poles situated halfway between the main poles and excited by a series winding so

proportioned as to furnish the amount of flux necessary to counteract the effect of the armature m.m.f.s in the zone of commutation. In machines furnished with commutating poles, the portion of the armature periphery covered by the main poles is usually limited to 0.62 or 0.65 of the total periphery.

FIG. 12.



In Fig. 10 the face of the commutating pole is supposed to cover the full length of the armature core and to be wound with sufficient ampere-turns to overcome the armature m.m.f. and force a certain amount of flux into the armature teeth comprised in the zone of commutation. In this case the flux required to balance or counteract the flux cut by the end connections of the short-circuited coil has been neglected, and the flux shown entering the teeth under the commutating pole is the slot flux only. If the face of the commutating pole were shorter than the axial length of the armature core (as is usually the case) there would also

be a certain amount of what has previously been referred to as the armature flux to be compensated for; but Fig. 10 has been drawn solely for the purpose of showing how the slot flux—which cannot be annulled like the armature flux—is supplied by the commutating pole, and since it no longer passes outward through the roots of the teeth, is not cut by the coil as it travels over the space of width W_a. When the conductors fill the slot instead of being concentrated at the bottom as shown in Fig. 10, a portion of the slot flux is cut by a portion of the conductors in the slot. It is therefore necessary to consider what shall be understood by the "equivalent" slot flux when this flux is supplied by the commutating pole (or leading pole tip when the brushes are shifted), because this will no longer have the same value as given by the previously developed formula (3).

The element of flux $d\phi$ (see Fig. 4) no longer links with $2T\frac{x}{d}$ conductors as when the slot flux was considered as passing outward through the roots of the teeth, but with $2T\frac{(d-x)}{d}$ conductors. The equivalent flux per centimetre axial length of slot when no part of this flux passes into the armature core below the teeth, is therefore

$$\Phi_{es} = 2\int_0^d d\phi \times \frac{d-x}{d}$$

$$= \frac{1.6\pi TI_c}{d^2 s}\int_0^d x(d-x)\,dx$$

$$= \frac{1.6\pi d}{6s}\,TI_c \qquad (8)$$

or just half the equivalent slot flux as given by formula (3), and one-third of the total slot flux as given by formula (2).

The question now arises: What is the necessary total flux entering the tops of the teeth comprised in the commutating zone to develop the proper voltage component in the short-circuited coil?

If the commutating pole is of the same length axially as the armature core, the flux cut by the coil side in the slot must be such as to generate an e.m.f. exactly equal but opposite to the e.m.f. developed by the cutting of the end flux ϕ_e, as this will produce

the desired condition of no change of flux through the coil during the time of commutation. If ϕ_d is the flux per centimetre of axial length passing into the armature through the roots of the teeth comprised in the zone of commutation of width W_a and length l_a, the total flux entering the armature teeth from the commutating pole is

$$\Phi_c = (\Phi_s + \Phi_d)\, l_a. \tag{9}$$

The condition necessary to produce the "neutral field" of commutation is

$$\Phi_e = (\Phi'_{es} + \Phi_d)\, l_a.$$

The quantity ϕ_s in equation (9) may be replaced by $3\phi'_{es}$ and a value for ϕ_d in terms of ϕ_e and ϕ'_{es} is obtained from the last equation. These substitutions lead to the equation

$$\Phi_c = 2\Phi'_{es} l_a + \Phi_e \tag{10}$$

or, if preferred

$$\Phi_c = \Phi_{es} l_a + \Phi_e. \tag{11}$$

Before discussing the more general case of an interpole of which the face does not extend the full length of the armature core, it will be advisable to group together and define clearly the various flux components which have to be considered.

ϕ_c = total flux entering armature teeth from interpole over area of width W_a and length l_p (see Fig. 11).

ϕ_e = end flux; being the amount of flux cut by the T_c conductors in the end connections of *one end* only of the armature.

ϕ_s = total slot flux per centimetre of armature length (two slots).

ϕ_{es} = equivalent slot flux per centimetre when magnetic lines pass outward from armature core through root of teeth (two slots).

ϕ'_{es} = equivalent slot flux per centimetre when magnetic lines pass inward from air gap through top of teeth (two slots).

ϕ_d = portion of interpole flux per centimetre length, which enters armature core through root of teeth.

ϕ_a = armature flux per centimetre length, which leaves teeth over the commutating zone of width W_a and length $l_a - l_p$ (Fig. 11).

The total amount of the armature flux is $\phi_a(l_a - l_p)$. It may be calculated by means of formula (4) or (5) which gives the quantity B_a, and ϕ_a is $B_a W_a$ where W_a is the width of the zone of commutation at the armature surface in centimetres.

Since the equivalent flux to be cut by conductors under the interpole must equal the total of all the flux components that have to be neutralized, we may write the equation

$$\phi_d l_p + \phi'_{es} l_p = \phi_e + \phi_a (l_a - l_p) + \phi_{es} (l_a - l_p).$$

The total flux in the air gap under the face of the commutating pole is

$$\phi_c = \phi_s l_p + \phi_d l_p.$$

Inserting for $\phi_d l_p$ in this last equation the value derived from the previous equation, we get

$$\phi_c = \phi_s l_p + \phi_e + \phi_a (l_a - l_p) + \phi_{es} (l_a - l_p) - \phi'_{es} l_p. \qquad (12)$$

This equation may be simplified by expressing the total slot flux ϕ_s and the equivalent slot flux ϕ'_{es} in terms of the equivalent slot flux ϕ_{es}. The relation between these quantities is obtained by comparing previously developed equations. Thus we have,

$$\phi_s = \frac{3}{2} \phi_{es}$$

and

$$\phi'_{es} = \frac{1}{2} \phi_{es}.$$

Inserting these values in equation (12), we get

$$\phi_c = \phi_e + \phi_{es} l_a + \phi_a (l_a - l_p) \qquad (13)$$

wherein the symbols ϕ_{es} and ϕ_a stand for the flux components *per centimetre length* of armature core, as previously defined.

Knowing the amount of flux to be provided by each commutating pole, its cross-section can be decided upon and the necessary exciting ampere-turns calculated, bearing in mind the following requirements:

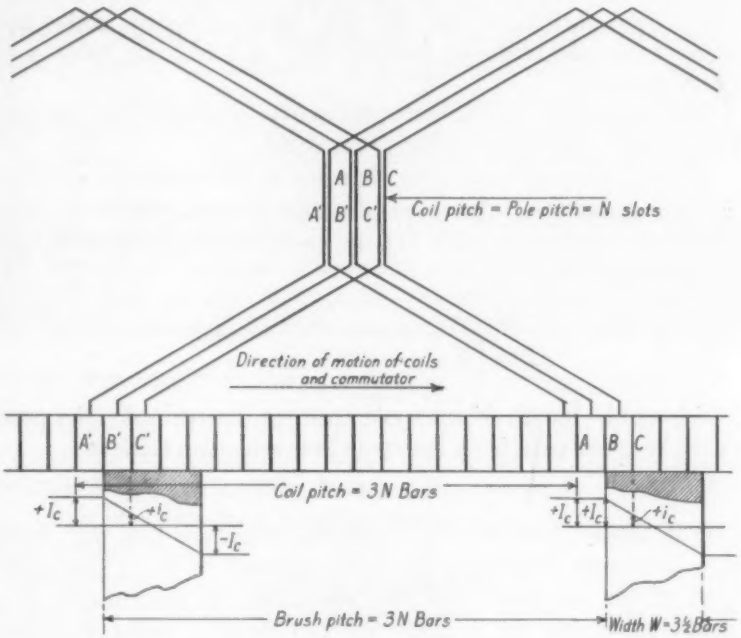
(*a*) The average air-gap density should be low (preferably not exceeding 4000 gausses with full load on the machine), to allow of increase on overloads.

(*b*) The leakage factor should be as small as possible. This involves keeping the width and axial length of interpole small, thus conflicting with condition (*a*) and presenting one of the difficulties of commutating pole design.

(*c*) The minimum width of pole face must be such that the equivalent pole arc (which includes an allowance for fringing) shall be somewhat wider than the commutating zone of width W_a.

(*d*) The effect of the interpole being to increase the flux in that portion of the yoke which lies between the interpole and the main pole of opposite polarity, it is important to see that the

FIG. 13.



resulting flux density in this part of the magnetic circuit is not excessive.

(*e*) The total line current should, if possible, pass through all the interpole windings in series; that is to say, parallel circuits should be avoided because of the possibility that the current may not be equally divided. If the total current is too great, a portion may be shunted through a diverter. The diverter should be partly inductive, the resistance being wound on an iron core in order that the time constants of the main and shunt circuits may be approximately equal. If this is not done, the interpole winding will not take its proper share of the total current when the change

of load is sudden, and this may lead to momentary destructive sparking.

Example of Commutating Pole Design.—In order to avoid unnecessary repetition, the data previously assumed for the calculation of the short-circuit e.m.f. will be used. With the same number of interpoles as there are main poles (*i.e.*, six in this example) the flux entering the teeth in the commutating zone of width $W_a = 3.5$ cm. should have the value given by formula (13). The end flux cut by the short-circuited coils has already been calculated by formula (6) and has the value $\phi_e =$ 43,200 maxwells.

The equivalent slot flux as previously calculated by formula (3) has the value $l_a\phi_{es} = 55{,}000$ maxwells.

The flux density, where the flux due to the armature currents leaves the armature in the space of width W_a and length $(l_a - l_p)$, as indicated by the arrows in Fig. 11, is calculated by formula (5) and has the value

$$B_a = \frac{960 \times 76}{7\left(1+\frac{2}{6}\right)(1-0\cdot625) \times 30} = 695 \text{ gausses.}$$

If l_p = axial length of interpole (not yet determined), the total flux which must pass from interpole into armature teeth is

$$\phi_c = \phi_e + \phi_{es} l_a + B_a W_a (l_a - l_p)$$

which is simply formula (13) with the armature flux per centimetre of length expressed as $B_a W_a$ instead of ϕ_a. The axial length of the interpole face can, therefore, be determined if a suitable value for the average air-gap density under full-load conditions is assumed. Let B_p stand for this value; then

$$B_p l_p W_a = \phi_e + \phi_{es} l_a + B_a W_a (l_a - l_p)$$

whence

$$l_p = \frac{\phi_e + \phi_{es} l_a + B_a W_a l_a}{W_a (B_p + B_a)}. \qquad (14)$$

With flux densities expressed in gausses, the dimensions must be in centimetres, and if B_p is taken as 4000 gausses, the length l_p of the interpole measured parallel with the axis of the armature is found by formula (14) to be almost exactly 10 cm. or (say) 4 inches.

The total flux in the air-gap under the commutating pole in the space of width $W_a = 1.375$ inches is

$$\phi_c = 43{,}000 + 55{,}000 + 695 \times 1.375\ (11 - 4)\ 6.45 = 141{,}400 \text{ maxwells.}$$

Flux in Core of Interpole.—Assuming the cross-section of the commutating pole to be as shown in Fig. 12, the total flux will have to be carried by an iron core of cross-sectional area $1.5 \times 4 = 6$ square inches. The air-gap is $\frac{3}{8}$ inch and the pole face is slightly smaller in width than the pole core; the effective or equivalent area (including an allowance for fringing) through which the flux enters the armature teeth will be something less than $2 \times 4 = 8$ square inches or (say) 6.5 square inches. Since we must have 141,400 maxwells entering the space of width $W_a = 1.375$ inches we require a total flux in the air-gap of $141{,}400 \times \frac{6.5}{1.375 \times 4} = 167{,}000$ maxwells. The leakage factor is always high because of the small distance between the commutating pole and main pole. If this leakage factor has the value 1.8, the total full-load flux in the core of the interpole is $167{,}000 \times 1.8 = 300{,}000$ and the flux density in the iron is $\frac{300{,}000}{6 \times 6.45} = 7750$ gausses.

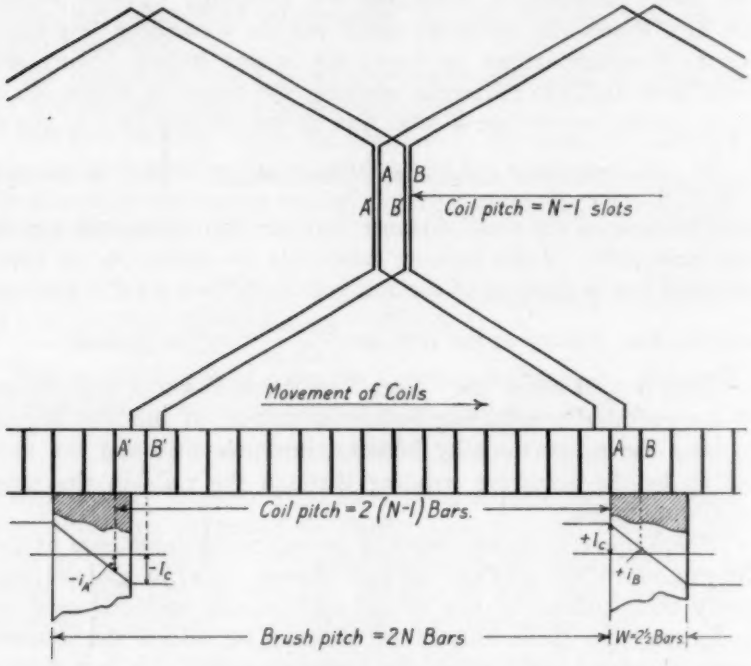
This is well below the "knee" of the B-H curve and allows of considerable overloading before saturation of the iron begins to limit the proportionality between interpole flux and the flux set up by the armature winding through the coils in the zone of commutation.

The ampere turns necessary to overcome the reluctance of the air-gap when the average air-gap density is 7750 gausses can easily be calculated by any of the approximate methods used by designers. To these ampere turns must be added the ampere turns to oppose and balance the armature m.m.f. It is not proposed to calculate the series winding required. A number of turns slightly in excess of the calculated number are put on the interpole, the final adjustment being made on test. A resistance—or diverter—may be used to shunt part of the current, or the strength of the reversing field may be adjusted by altering the reluctance of the magnetic circuit of the interpole. This may be accomplished without changing the length of the air-gap between pole face and armature by inserting non-magnetic distance pieces between yoke ring and commutating pole. If this

method is adopted the machine would be designed with two or three iron packing pieces clamped between the pole core and yoke. One or more of these would be removed and replaced by brass plates if it were found desirable to reduce the amount of the reversing flux.

Methods of Reducing Flux Pulsations under Commutating Poles.—Although the fixed position of the brushes relatively to

FIG. 14.

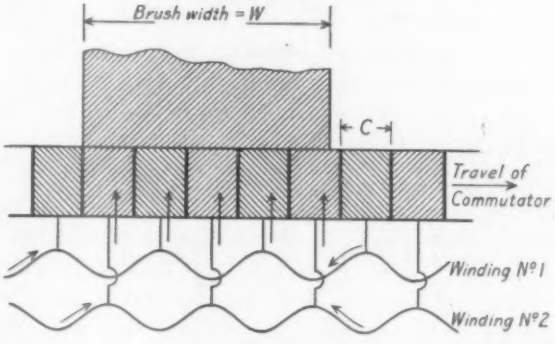


the field poles maintains the fluxes due to the armature windings generally in a fixed position in space, there will be certain oscillations and pulsations of flux which it is necessary to keep within limits that can be taken care of by the contact resistance of carbon brushes.

The amount of the flux oscillations under the interpole will depend upon the relative dimensions of pole face and tooth pitch and upon the angular width of the commutator bars, which should be kept small. By proportioning the face of the commutating pole

so that the reluctance of the air-gap varies as little as possible, whether a tooth or slot occupies the position under the centre of the pole, the flux pulsations due to variations in the reluctance of the magnetic circuit may be kept within reasonable limits. Obviously, a large tooth pitch is to be avoided, and the length of the air-gap under the interpole should be determined with this question of flux pulsations in mind. A narrow pole face and comparatively large air-gap will generally lead to good results.

FIG. 15.



It is usually advisable to make the air-gap under the commutating pole about 25 per cent. longer than the gap under the centre of the main poles.

It has already been shown that the ampere turns on the commutating pole must be somewhat greater than the armature ampere turns which must be opposed and annulled before any flux can enter the armature from the commutating pole. In order to ensure equality of strength, the windings of the interpoles are usually all connected in series, which suggests that the armature m.m.f. is assumed to be of exactly the same amount under each interpole. With the series or two-circuit armature winding this condition is necessarily fulfilled, but with parallel windings the armature currents may not be perfectly balanced in the various circuits. That is the reason why series armature windings are generally to be preferred if there is likely to be any difficulty with commutation, but when proper provision is made for equalizing the currents in the armature circuits by means of a large number of equalizing connections satisfactory commutation may be obtained even with large parallel-wound armatures.

Effect of Several Coil-sides in One Slot—Full-pitch Windings.—When developing formulas (2) and (3) for the slot flux and equivalent slot flux, two assumptions were made: (1) That the number of commutator segments was the same as the number of armature slots, and (2) that the coil pitch was the same as the pole pitch. When there are several coil sides per slot, which occurs when the number of commutator segments is a multiple of the number of slots, the current at the beginning and end of the commutation period has not the maximum value $+I_c$ and $-I_c$ in all the conductors occupying the slot for which it is desired to calculate the leakage flux. The slot leakage flux is then not correctly given by formulas (2) and (3), and, for a closer approximation to actual conditions, a modifying factor should therefore be introduced. In other words, a mean value for the current in the slot which sets up the leakage flux should be substituted for the maximum value I_c of the formulas (2) and (3).

By drawing a diagram such as Fig. 13, and assuming the change of current in the coils undergoing commutation to follow the "straight-line" law, it is an easy matter to calculate the required correcting factor. Fig. 13 illustrates the condition of a dynamo with full-pitch multiple winding, but with three times as many commutator bars as there are slots. There are six coil sides in the slot, the sides A, B, and C being (say) in the upper half, while the sides A', B', and C' are (say) in the lower half. The brush covers $3\frac{1}{2}$ commutator segments and the diagram shows the condition existing just as the coil B is about to be short-circuited by the right-hand brush at the beginning of the period of commutation. The average current in the slot conductors tending to set up leakage flux is seen to be no longer I_c but $\frac{4\,I_c + 2\,i_c}{6}$.

The value of i_c is $I_c - I_c\left(\frac{2}{3{\cdot}5}\right)$, which gives for the average current per conductor at the beginning of commutation of the coil B the value 0.81 I_c. A similar calculation may be made for the coil B at the end of commutation at the instant when the mica B passes out from under the brush, and the remaining coils in the slot may be dealt with in a similar manner. The writer proposes the following formula to facilitate the calculations. If m

is the multiplier or correction coefficient by which the current I_c or the slot fluxes ϕ_s and ϕ_{cs} of formulas (2) and (3) must be multiplied, we have:

$$m = 1 - \frac{k-1}{2b} \qquad (15)^8$$

for full-pitch coils, where k = number of commutator bars per slot and b = number of bars covered by brush.

Formula (15) may be used whenever b is greater than k. For the special case of a very narrow brush when b is equal to or less than k, use the formula

$$m = \frac{b+1}{2k} . \qquad (16)$$

Effect of Short-pitch Winding on Slot Leakage Flux.— Although chord windings will usually slightly reduce the amount of armature flux in the zone of commutation and also the end flux, it is in the amount of the slot flux that their effect is most noticeable.

Fig. 14 is a diagrammatic representation of a chorded winding with the coil pitch one slot pitch shorter than the pole pitch, and with twice as many commutator bars as there are slots. There are four coil sides to the slot, and the figure shows the condition when the coil A is just about to be short-circuited by the brush, which covers 2½ commutator segments. The average value of the current per armature conductor under these conditions is seen to be $\frac{1}{4}\left[I_c+i_B-I_c-i_A\right]$, a quantity which can be calculated on the assumption of the "straight-line" law of current reversal.

The formula proposed by the writer for a correction factor to take care of chorded windings, together with any number of coil sides per slot, is

$$m' = m\left[\frac{1}{2} + \frac{b-k(s-1)}{2(b+k)}\right] \qquad (17)$$

where m, b, and k have already been defined, and s is the number

[8] When k is greater than 2 a more correct formula for the average of all the coils in the slot is

$$m = 1 - \frac{5}{12}\left(\frac{k-1}{b}\right)$$

but it is suggested that the simpler formula (15) is sufficient for the needs of the designer.

of slots chorded, or the difference between the number of slots in the pole pitch and the number of slots spanned by the armature coil. For the winding illustrated by Fig. 14 we have $s = 1$. Formula (17) may be used for any value of s between $s = 0$ and $s = \frac{b}{k} + 1$.

For special case of a very narrow brush, when b is equal to, or less than, k, the formula is

$$m = \left(\frac{b+1}{2k}\right)\left(0.5 + \frac{2-s}{4}\right) \qquad (18)$$

which may be used for values of s between 0 and 2.

Numerical Example Illustrating Use of Formulas for Correction Factors.—In a previous example with slot dimensions $d = 1.5$ inch, $s = 0.39$ inch, and 8 conductors in the slot each carrying a maximum current of 76 amperes, the equivalent slot flux was found to be $\phi_{es} = 55{,}000$ maxwells. This was for a full-pitch winding with the same number of commutator bars as slots, namely 20 slots per pole and 20 commutator segments between + and − brush sets. Let us now calculate the slot flux on the basis of the same total ampere conductors per slot, namely $8 \times 76 = 608$, but with only two turns per coil (*i.e.*, twice as many commutator bars as there are slots), a brush covering 3 commutator segments, and a chorded winding with the coil spanning 18 slots. The quantities for use in the formulas (15) and (17) are then $k = 2$, $b = 3$, and $s = 2$, and

$$m' = \left(1 - \frac{k-1}{2b}\right)\left[\frac{1}{2} + \frac{b - k(s-1)}{2(b+k)}\right]$$

$$= \left(1 - \frac{2-1}{6}\right)\left[\frac{1}{2} + \frac{3 - 2(2-1)}{2(3+2)}\right]$$

$$= 0.5.$$

Whence the equivalent slot flux is no longer 55,000 maxwells as in the previous example, but half this amount, or 27,500 maxwells, which indicates that the amount of flux from interpole or leading pole tip to produce sparkless commutation will be less than with the full-pitch coils and the smaller number of commutator segments.

Duplex Windings.—In machines of very large current capacity the armature is sometimes wound with two circuits

connected in parallel by the brushes. The connections from any one winding to the commutator are then taken to alternate commutator segments, leaving the intermediate segments for connection to the other winding, all as indicated by the diagram, Fig. 15. The study of commutation in the rare cases when such double windings are used may be made on the same basis as for simplex windings, and the formulas (3), (15), and (17) may be used for slot flux calculations. Thus the reversing flux from the interpoles necessary to produce ideal commutating conditions will be approximately the same whether we have a simplex winding with two conductors per slot each carrying I_c amperes, or a duplex winding with four conductors per slot each carrying $\frac{1}{2}I_c$ amperes, and twice the number of commutator segments.

It should, however, be noted that the travel of the conductor on the armature periphery is no longer $W_a = W\frac{D}{D_c}$ which is a good enough definition of W_a for a simplex winding when the mica insulation between commutator segments is of negligible thickness. With the duplex winding it will be evident from a study of the diagram, Fig. 15, that we now have $W_a = (W - C)\frac{D}{D_c}$, where W = brush width, C = pitch of commutator bars, D = diameter of armature, and D_c = diameter of commutator.

For the same reason, namely, that the space between commutator segments of any one winding is the width of the bar pitch instead of merely the thickness of the mica, the time of commutation is

$$t_c = \frac{W - C}{V_c}$$

where V_c is the surface velocity of the commutator.

If the substitution of a duplex winding for a simplex winding involves not only the subdivision of the armature winding into two separate circuits connected in parallel by the brushes, but also the doubling of the number of slots, so that the ampere-conductors per slot are reduced to one-half what they would have to be with a simplex winding, it is obvious that the slot leakage flux may be considerably reduced, although little or no change will take place in the armature flux (or the density B_a of formula 4 or 5) and the end flux (formula 6).

Commutating Field Obtained by Brush Shift.—Little need be

said about commutation by moving the brushes in order to find under the main poles a magnetic field which will bring about proper reversal of the current in the commutated coils, because this method is little used at the present day. Graphical methods may be used to obtain a very close approximation to the true distribution of flux density in the air-gap under load conditions, and the problem then resolves itself into finding the brush position which will place the short-circuited coil in a reversing field of such strength as will satisfy the condition indicated by formula (11), which applies to an interpole covering the full length of the armature core.

Pole Face Windings.—It is not proposed to discuss the advantages and disadvantages of pole face windings, the main purpose of which is to compensate for field distortion due to the currents in the armature conductors. With brushes on the geometric neutral as in interpole machines, field distortion has very little to do with commutation; but obviously if a winding is provided in slots through the pole shoes to annul the cross-magnetizing ampere turns of the armature winding, the amount of winding necessary on the commutating poles will be greatly reduced. The problem of interpole design remains nevertheless practically unchanged because, if sparkless commutation is to be obtained, the amount of flux entering the armature in the zone of commutation must be the same whether or not pole face windings are used.

One advantage of compensating or pole face windings is the improvement of the "field form" and the avoidance of local flux concentration in the air-gap when the machine is heavily loaded. This greatly reduces the tendency to flash over at the commutator surface, a trouble which has very little connection with the reversal of current in the coil undergoing commutation.

Concluding Remarks.—Since this article deals mainly with the principles underlying the phenomena of commutation, many practical considerations which bear upon sparkless commutation have not been discussed. It is obvious, for instance, that the nature and peripheral velocity of the commutator surface, the hardness of the mica insulation, the quality of the carbon brushes, and many other factors of a similar nature must necessarily be considered in a complete study of the phenomena of commutation, and the designer is also interested in the mechanical construction and temperature rise of the commutator. The predetermination

of the load which a new design of machine will carry without injurious sparking is admittedly a difficult problem, and although formulas have been developed and their use explained, it is well to bear in mind that no general formula or method of attack will solve all problems of commutation. Special cases require special treatment, but once the underlying principles have been clearly recognized, such minor modifications in the formulas and calculations as may be necessary to suit individual cases can usually be made without involving any radical changes in the method of procedure.

On the Separation of the Isotopes of Mercury. J. N. BRONSTED and G. HEVESY. (*Phil. Mag.*, Jan., 1922.)—By his brilliant experiments Aston has laid a firm foundation for the belief that many substances hitherto regarded as elements are in reality not elements at all but are mixtures of isotopes, that is, of genuine elements whose atomic weights are not far apart and whose chemical properties are identical, or at least so nearly so that chemical methods are incompetent to separate the constituent isotopes. Mercury is such a substance, and these two investigators in the Polytechnic Institute of Copenhagen have addressed themselves to the task of finding whether pure, ordinary mercury can be divided into portions having a difference in some physical property. "Only a few methods of separation come into consideration—chiefly, those which make use of the difference in the molecular velocities (atomic velocities) appearing as a consequence of the mass difference of the isotopes." Two methods are selected for trial, those of evaporation and of effusion. If a liquid, made up of two isotopes, is in equilibrium with its vapor above it, then as many molecules of the first isotope escape from the liquid into the vapor per second as are changed from vapor into liquid in the same time by striking against the liquid and being there held fast. The same condition of equilibrium holds for the second isotope, but it would be wrong to conclude that in a second just as many molecules of the first isotope as of the second pass from the liquid into the vapor or that equal numbers of the two isotopes change in equal times from the vapor to the liquid phase. On the contrary, since the kinetic energy of a molecule of one isotope in vapor equals the kinetic energy of a molecule of the other in the same state and since, further, the masses of the two molecules are different, it follows that the molecule of the lighter constituent must move with a greater velocity. More molecules of this vapor will, therefore, impinge on the surface of the liquid in a given time, more will fail to get away, and, to maintain equilibrium, more of the same kind must escape from the liquid into the vapor. How can use be made of this difference to effect any separation of the isotopes? "This is

most easily accomplished by allowing the liquid to evaporate in a vacuum and placing a highly-cooled glass plate over its surface. Now when the vapor pressure of the liquid is sufficiently slight, each molecule which leaves the liquid will reach the cooled wall before it has had an opportunity of meeting other molecules and being thrown back into the liquid. Having reached the cooled wall, it will be held by it, transferred into the solid state, and hindered in re-evaporation." The mercury for the experiment was placed in the space between the inner and the outer flask of a Dewar bulb, which was well evacuated. Liquid air was put in the inner flask and the bulb was set in an oil bath at 40°-60°. The distance from the liquid mercury to the cold glass surface was from 1 to 2 cm. The mercury slowly evaporated and turned into a solid upon the glass. Later this distilled portion was separated from the residual liquid and the densities could then be determined, for density was the physical property in which the differences were sought. The residual mercury was treated again and again in the same manner until of the original 2704 c. cm. only .2 c. cm. had not evaporated. The densities were found to grow steadily greater as the process went on. Calling the density of the original mercury unity, that of the second residual was 1.000016; of the twelfth, 1.000079; and of the eighteenth, 1.00023. On the other hand, when the first distillate was subjected to a series of further distillations it became manifest that the density grew smaller and smaller. The first distillate had a value of .999977; the fifth, .999881; the fourteenth, .99974. The greatest density found seems to have been 1.00023 and the smallest, .99974, or a difference of 49 parts in 100,000, "corresponding to a difference of 0.1 unit in the element weight of mercury." (Element weight means the mean atomic weight of the mix-element.)

The second method of separation consists in heating mercury and letting the resulting vapor flow out and be condensed in two similar cold glass tubes. The vapor comes first to a tube whose orifice is covered with foil pierced with numerous tiny holes. The faster molecules of vapor will find their way in greater numbers through these holes so that the vapor which passes on and is later condensed in the second tube will be richer in the slower and heavier isotope. It appears that only a few experiments were made in this manner. The density of the liquid collected in the first tube was .999987, thus confirming the previous results.

The authors conclude "Our results agree with those of Aston's, found by means of his mass-spectrograph, in the fact that Aston found a strong line corresponding to the atomic weight 202 and an undissolved band the centre of which corresponds to about 199. A very slight quantity of an isotope with the atomic weight of 204, the presence of which follows from Aston's experiments, cannot be found by our method if the separation is not to be pursued very considerably farther." G. F. S.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

THE FIELD RADIATED FROM TWO HORIZONTAL COILS.[1]

By G. Breit.

[ABSTRACT.]

In order to facilitate the landing of airplanes in times of poor visibility, Mr. J. A. Willoughby of the Bureau of Standards has suggested a special type of transmitting coil antenna.

This consists of two equal horizontal coaxial coils, one placed above the other. Both coils carry equal currents, and the current in one coil flows in a direction opposite to that of the current in the other.

It was believed that the waves would radiate in the form of a beam or inverted cone above the antenna. Experiments showed this to be true. The paper here abstracted concerns itself with the theoretical side of the question. In this paper the radiation from Mr. Willoughby's transmitting coil antenna is calculated.

The method of calculation is that of retarded potentials used in the form employed by Lorentz. The radiation is calculated for two cases, namely that of a circular coil and that of a rectangular coil.

By this method the value of the electric intensity is derived for points at distances from the transmitting coil antenna such as are of interest in actual practice.

It is found that if the effect of the earth is negligible, then an airplane carrying on its wings a receiving coil having its plane vertical receives a maximum of signal when it is situated in such a position that the line joining it to the transmitting coil antenna makes an angle of 30° with the vertical. The signal vanishes when the airplane is directly over the transmitting antenna and it vanishes rather soon after the airplane passes over the region of maximum signal and flies away from the transmitting antenna.

The effect on the transmission of having a perfectly conducting earth directly under the transmitting antenna has been investigated

* Communicated by the Director.

[1] Scientific Papers No. 431.

and it has been found that in this case a maximum signal is obtained when the line joining the airplane to the transmitting coil antenna makes 26° 34′ with the vertical.

In both cases the region of space within which the signal can be detected by receiving instruments of given sensibility has nearly the form of the space between two inverted coaxial vertical cones of finite length having their common apex at the transmitting station.

WEIGHING BY SUBSTITUTION.[2]

By C. A. Briggs and E. D. Gordon.

[ABSTRACT.]

THE basic principle of " substitution weighing " is very simple. In general, the object to be weighed is placed on the platform of a scale and balanced. This load is then removed, and standard weights are substituted until the balance is restored. The standard weights needed for this are equal to the required weight of the object. The method has the important advantage that the effect of certain errors in the scale, otherwise affecting the results, can be eliminated. By observing certain precautions and details in the procedure very accurate results are possible, and this subject is covered in the present work.

This paper describes a plan for making substitution weighings which can be applied in using either equal-arm balances or compound lever scales. It is based upon the method developed by the Bureau of Standards in the test and standardization of the 10,000 pound weights which form a part of its railroad track scale testing equipments and in which a maximum accuracy is desired. The interest of the practical scale man coming in contact with the work of the Bureau has been aroused in its methods, and the present publication is prepared mainly in response to a general demand for an outline of a systematic method for carrying out substitution weighing. As the result of suggestions of these men engaged in the actual maintenance and testing scales, the procedure for taking data and conducting the routine of the test has been made to conform closely with that used by the Bureau of Standards in the precision test of railroad track and grain hopper scales; and the form for recording the data and making computations

[2] Technologic Papers No. 208.

which is presented has been arranged to be as near to that used in the scale-testing work as is practical.

It should be clearly understood that while the method was developed in connection with the testing of large weights, one of the fields of greatest usefulness is in the careful test of ordinary sized weights forming the working standards used by those testing scales and also for the test of counterpoise weights.

It is also believed that the publication will serve as a very useful reference for almost all industrial, engineering, or other technical laboratories where there is an occasional need for making weighings with more than ordinary accuracy. The paper offers a plan for undertaking such work promptly, as the steps to be taken and the record form and computation sheet to be used are presented in detail.

In the method of weighing by substitution presented in this publication readings are taken from the moving beam by means of a pointer and scale which are applied to the mechanism. The accuracy obtained is not limited by any error in the multiplication ratios of the weighing mechanism, but only by the precision with which it will repeat its indication.

The general matters covered by the paper comprise first a general description of the theory of weighing by substitution; the preparation of the scale for weighing so that the swings of the beam can be read on the graduated scale; the method for obtaining the positions of the equilibrium of the beam from the readings taken on it while moving; the method of removing and substituting weights; and a description of the details to be observed in preparing the scale and making observations, and the practice to be followed in making the computations.

The method is of especial value in the calibration of a large number of weights of the same denomination. The method is equally applicable, however, for determining accurately the unknown weight of any object.

A Study of Franklin's Experiment on the Leyden Jar with Movable Coatings. G. L. Addenbrooke. (*Phil. Mag.*, March, 1922.)—The usual form of this experiment is as follows: A glass jar has an inner and an outer metallic coating, both closely fitting and capable of being removed. This jar is charged in the regular way. The inner coating is removed by an insulated holder. It then has

scarcely any charge. It is touched to the outer coating which may be still in place or may have been separated from the glass. The two coatings are then put into position and it is found that the reconstituted jar has a strong charge. This is interpreted to mean that the seat of the charge was in the dielectric and that the charge remained there when the jar was taken apart.

The author of the paper could not reconcile the explanation with his views of electrical actions. He noted that glass was always mentioned as the material of the jar. He tried a jar of paraffin and found marked differences. Upon removal the inner coating had an electric charge. After the coatings had been touched and the jar put together, there was no appreciable charge left. Exactly the same results were obtained when precautions were taken to have the glass coatings free from moisture. It thus appears that in the experiment, as usually performed with glass as the dielectric, a damp layer on each side of the glass really serves as a coating. The removal of the coatings is, therefore, fictitious. The inner metal coating brings away with it only a little charge, the main part of the charge remaining behind on the damp layer.

It is suggested that the experiment be performed with the substitution of an ebonite or of a glass jar. The former should be kept in the dark with calcium chloride. The latter should be heated to 100° C. and kept with the hygroscopic substance as long as possible.

G. F. S.

Natural Gas in 1919. (*U. S. Geological Survey Press Notice.*)—The production of natural gas in the United States in 1919 was 11.8 per cent. more than in 1918. The figures for 1919, however, are not directly comparable with those for 1918, because the production in 1919 includes the volume of gas wasted in the fields and in transmission, as far as reported by the operators. The quantity of gas wasted is the difference between the quantity produced and that actually consumed, a difference of 58,943,631,000 cubic feet. As the figures for 1919 include this wasted gas, no value is given to the quantity produced. If the volume of gas wasted had been utilized it would have possessed a value of about $13,725,000. This quantity does not include the gas wasted at the point of consumption. The quantity produced in 1919, not including the gas wasted, is therefore the quantity consumed, which was 739,915,781,000 cubic feet, an increase of only 2.6 per cent. over 1918.

The total acreage reported in 1919 is considerably less than in 1918 and previous years because it includes only the acreage containing gas wells and that where gas wells predominate. The figures for past years included much of the oil acreage. It is impossible to separate the natural gas and petroleum acreage.

NOTES FROM THE RESEARCH LABORATORY EASTMAN KODAK COMPANY.*

ON CONVECTION EFFECTS IN PHOTOGRAPHIC BATHING OPERATIONS IN THE ABSENCE OF AGITATION.[1]

By E. R. Bullock.

THE relatively more rapid bleaching of a developed silver image on cinematograph film at the lower end of a strip immersed vertically in a ferricyanide-iodide solution was found to be a special case of a general phenomenon, which is seen with all silver images in gelatin or collodion on an impervious support when immersed vertically in any halidizing (halogenizing) bleach. By theoretical reasoning it was concluded that the phenomenon must be attributed to the effect of the existence of an upward convection current having its origin in the diminution of density of the bleach solution in contact with the image by the chemical reaction which takes place. Silver-dissolving solutions were found to act more rapidly at the upper than at the lower end of a vertically immersed silver image. The actual existence of upward convection currents during the action of halidizing bleaches, and of downward currents with silver-dissolving solutions, was demonstrated by the motions of suspended short lengths of cotton fibre. (A general method is thus available for the demonstration of the occurrence of a chemical reaction between a solid and a liquid. As an example, it was found that silver bromide reacts with a dilute potassium selenocyanide solution, and from this fact, with others, it was concluded that silver selenocyanide is less soluble than silver bromide, but more soluble than silver iodide.) In ordinary stand development (with vertical immersion and without agitation) a downward current prevails, and the rate of development is accordingly somewhat greater near the top than near the bottom of a negative. Under usual conditions the density difference due

* Communicated by the Director.

[1] Communication No. 33 from the Research Laboratory, Eastman Kodak Company, and published in *Phot. Amer.*, March, 1922, p. 162, and *Brit. J. Phot.*, Feb. 24, 1922, p. 110.

to this cause is of the order of magnitude of 5 or 10 per cent. Convection currents during the fixing of bromo-iodide emulsions seem to harmonize with the view that, broadly speaking, the silver bromide dissolves before the silver iodide. It is probable that immediately after the disappearance of the last traces of opacity during fixing, the whole of the silver has been converted into the very soluble double thiosulfate.

Active Modifications of Hydrogen and Nitrogen Produced by Alpha Rays. F. H. NEWMAN. (*Phil. Mag.*, March, 1922.)—When an electric discharge passes through nitrogen or hydrogen it is found that these gases are absorbed when certain elements are present in the tube. At least in part the absorption is due to chemical action and seems to be caused by the production of active modifications of the gases. The present Lord Rayleigh found nitrogen drawn from the discharge tube to possess active properties and concluded that the active form was atomic in composition. Wendt has shown that the electric discharge in hydrogen at low pressures develops a modification of that element in which the molecule consists of three atoms. On the other hand, it is known that in a general way the chemical effects of alpha rays resemble those of the silent discharge. It seems, therefore, not unlikely that nitrogen and hydrogen under the action of these rays would comport themselves as they do when traversed by the electric discharge. The experiment has been tried by Mr. Newman and he finds that nitrogen after bombardment by alpha rays from polonium is little by little absorbed when sodium, potassium, sodium-potassium alloy, sulphur, phosphorus, iodine, arsenic, magnesium or mercury are present in the tube. The disappearance of the gas could not be due to occlusion, for upon heating it was not set free. This also indicates that any compounds formed are quite stable. After hydrogen had been treated in like manner it, too, was absorbed by sodium, phosphorus or iodine. Upon heating, however, it was re-liberated, indicating either occlusion or the formation of unstable compounds. A chemical test proved hydrogen sulphide to be formed when sulphur was in the hydrogen tube.

Neither ultra-violet light nor the radiation from polonium exclusive of alpha rays was competent to produce the same results as those achieved by the alpha rays. G. F. S.

NOTES FROM THE U. S. BUREAU OF CHEMISTRY.*

CRYSTALLINE CHLORO-TETRA-ACETYL MANNOSE.[1]

By D. H. Brauns.

[ABSTRACT.]

CHLORO-TETRA-ACETYL mannose, hitherto known only in the form of a syrup, has been obtained in crystalline form.

The behavior of the crystalline substance toward methyl alcohol and silver carbonate (reaction of Koenigs and Knorr) is analogous to that of bromo-tri-acetyl rhamnose. This is well accounted for by their structural formulas.

In the case of chloro-tetra-acetyl mannose, this reaction leads to various methyl tetra-acetyl derivatives and in this respect chloro-tetra-acetyl mannose differs from the corresponding derivatives of glucose and galactose, which yield only one methyl tetra-acetyl derivative.

This difference in behavior may lead to a more detailed knowledge of the arrangement of the constituent parts of their molecules.

OBSERVATIONS UPON THE RESISTANCE OF THE RAT TO CONSECUTIVE INJECTIONS OF STRYCHNINE.[2]

By E. W. Schwartze.

[ABSTRACT.]

THE tolerance of the rat to repeated subcutaneous injections of strychnine has been studied. Evidence has been adduced to show that the coefficients of disposal which are necessarily expressed in terms of percentage of minimum lethal dose per given period of two hours are too low. They should be reinterpreted to fit the case of oral administration in which large amounts are administered at once and in which type of experimentation the absorption of strychnine has an opportunity to keep pace with the disposal. Accordingly, the coefficient of disposal has been

* Communicated by the Chief of the Bureau.

[1] Published in *J. Am. Chem. Soc.*, **44** (1922), 401.

[2] Published in *J. Pharmacol.*, **19** (1922), 49.

regarded as constant and the absolute amount disposed of as a variable depending on the amount present. On this basis the disposal of strychnine by the rat may reach at least 1 mgm. per kilo per hour. This occurs only when the strychninization is kept constantly very close to the maximum limit, by absorption from the gastro-intestinal tract.

The coefficients as determined arithmetically have been checked by a new experimental criterion, namely, the relation of survivals to fatalities in a given uniformly treated series and comparison of one such series with another.

The extremely high tolerance of the rat (as well as of other animals) to consecutive injections of strychnine would seem to be significant with respect to the possibility of correlating this with the failure to demonstrate as yet an habituation to this drug.

BOTULISM FROM THE REGULATORY POINT OF VIEW.[3]

By Charles Thom.

[ABSTRACT.]

TRANSFORMED into concrete suggestions, responsibility for food poisoning, in so far as canned goods are concerned, rests about equally upon the dealer and upon the household. Every dealer should be made to feel a direct and positive obligation to remove from sale any can of food which shows sign of spoilage, and to condemn as dangerous and return to the packer any lot in which spoilage appears in considerable percentage. Correspondingly, some one, at least, in every household, should definitely understand the dangers of spoiled food, canned or uncanned, make intelligent examination of all food bought and apply discriminating care to all food stored in the home.

As a basis for propaganda to eliminate the danger of poisoning cases, these recommendations may be given the widest publicity.

(1) Clean, fresh, sound food will not cause botulism.

(2) Food freshly heated to the boiling point will not cause botulism.

(3) Recooking is advised whenever opportunity for microbial activity has been given, but only before spoilage becomes evident by appearance, odor or taste. Food showing signs of decom-

[3] Published in *Am. J. Public Health*, 12 (1922), 49.

position is potentially dangerous; hence physical evidence of spoilage should lead to destruction, not to salvaging.

(4) Intelligent coöperation of packer, dealer, and householder in the examination of food before it reaches the table is essential to the prevention of botulism.

RATIONS FOR FEEDING POULTRY FOR THE PACKING HOUSE.[4]

Contribution from the Food Research Laboratory.

[ABSTRACT.]

EXTENSIVE feeding experiments on both a laboratory and commercial scale showed that a great variety of feeds can be used in fattening broilers, springs, roasters, and hens. Numerous analyses of unfed and fed birds are recorded. Data are given on the weight lost by range and fed birds during dressing, on the percentage increase in weight of the different classes of birds receiving the experimental rations for fourteen days and of those fed the control ration of cornmeal (forty parts) and buttermilk (sixty parts) for four, eight, eleven and fourteen days, and on the character of gains made in fourteen days by the use of the control ration. The experiments show clearly that the growing birds require a different ration from that of the adult fowl. The merits of different feeds from the standpoint of their content in protein, fats, carbohydrates, salts, and vitamines are discussed.

Demonstration of the Protective Effect of a Colloid.—Prof. J. Newton Friend communicates to *Nature* the following experiment showing the effect of a colloid on chemical change. This is afforded by the precipitation of mercuric iodide on addition of the chloride to potassium iodide. If this is effected in fairly dilute aqueous solution, the unstable yellow form is first precipitated and rapidly turns from orange to red as it becomes converted to the more stable variety. If, however, the reaction is carried out in the presence of gelatin, say, one per cent., the liquid first turns momentarily yellow, due to the formation of colloidal mercuric iodide, then becomes turbid, and a beautiful canary color develops, which remains practically unchanged for half an hour or more, according to circumstances. Only very slowly does it change to the red polymorph. The protective colloid retards the growth of the yellow particles. Sunlight accelerates the

[4] *U. S. Department of Agriculture Bulletin*, issued March 13, 1922.

change markedly. With the aid of the ultra-microscope (one-half inch oil immersion) these changes may be seen clearly. Drops of gelatin and dilute potassium iodide are mixed under the coverglass and the ultra-microscope focussed as usual. A drop of mercuric chloride solution is brought to the edge of the coverglass and is drawn under by capillary action. The field of the ultra-microscope becomes swept with a stream of luminous particles moving with dazzling velocity—the Brownian movement of the colloidal mercuric iodide. The velocity slows down as the particles increase in size, until the colloid range has been passed, and in a few minutes a fine precipitate is obtained evincing scarcely any movement.

H. L.

The Theory of the Intrinsic Field of a Magnet and the Relation of Its Magnetic to Its Characteristic Electric and Thermal Properties. J. R. ASHWORTH. (*Phil. Mag.*, March, 1922.)—For the explanation of the phenomena of ferro-magnetism, Pierre Weiss and others have assumed the existence of a magnetic field of force within the magnetic material. If the value of this intrinsic field be calculated from the relations between magnetic intensity and temperature, it appears to have an enormous value, while, if, on the other hand, it be derived from the effects of the application of an external magnetizing field, it comes out quite small. The large value has been in favor because it conveniently explained certain effects.

When ferro-magnetic substances are heated to the critical temperature, they lose their remarkable magnetic qualities and degenerate into mere paramagnetic materials. At about the same temperature there are likewise notable modifications in their electric resistivity, thermo-electric power and specific heat. It seems a fair assumption that the changes in these three properties are bound up with the disappearance of the intrinsic field at the critical temperature and, further, could the critical temperature with the accompanying loss of the intrinsic field be by some means shifted to some other temperature then these three properties also should show changes in the temperatures at which they manifest themselves, and this all the more if the intrinsic field is very strong. Happily there is a way of shifting the critical temperature in the case of nickel. The application of an alternating field to this metal reduces the critical temperature by from 50 to 100 degrees C. Experiments were made, and it was found that there was no corresponding displacement in the temperatures at which changes in the three properties occur. This indicates, though it does not prove, that the intrinsic field obliterated at the critical temperature is not so large as has been thought.

The author regards the intrinsic field as made up of two constituents, a molecular field of great strength and a magnetic field of small strength. Such a composite field has the advantage of explaining quite a number of magnetic effects. G. F. S.

NOTES FROM THE U. S. BUREAU OF MINES.*

COMPARISON OF TWO METHODS OF DETERMINING FUSIBILITY OF COAL ASH.

By A. C. Fieldner, W. A. Selvig and W. L. Parker.

THE micropyrometer method for determining coal-ash fusibility is more rapid than the gas-furnace method and is more comfortable to the operator. Coal ashes fusing under 2600 degrees F. by the gas-furnace method can in the majority of cases be checked within 100 degrees F. by the micropyrometer method if fused in a reducing atmosphere of combustion gases similar to that employed in the gas-furnace method. Very refractory ashes, fusing above 2800 degrees F. as determined by the gas-furnace method, tend to give considerably lower results by the micropyrometer method. The two methods can therefore not be considered as strictly alternate methods for all ashes. The great majority of coal ashes from American coals, however, fuse below 2800 degrees F. in the gas-furnace.

This subject is discussed in greater detail in a paper presented before the meeting of the American Chemical Society, in April, at Birmingham, Alabama.

TESTS OF HAND EXTINGUISHERS ON ZINC-DUST FIRES.

By S. H. Katz and J. J. Bloomfield.

FIRES in zinc dust sometimes give particular trouble, owing to their resistance to extinguishing, and the liability of explosions. Tests were made on commercial zinc dust with various extinguishers. Water aggravated the fire and increased the danger, as did sodium bicarbonate solution. Carbon tetrachloride, and silicon tetrachloride produced dense smoke. A " frothy mixture " was the best, smothering the flame quietly and effectively. Further details are given in a paper issued by the Bureau in mimeograph form.

* Communicated by the Director.

PHYSIOLOGICAL EFFECTS OF EXPOSURE TO LOW CONCENTRATIONS OF CARBON MONOXIDE.

By R. R. Sayers, F. V. Meriwether and W. P. Yant.

THE effect of comparatively low concentration of carbon monoxide for short periods and under normal air conditions of temperature and humidity, with the subject at rest, was studied by Dr. Yandell Henderson and his co-workers. In making recommendations to the New York and New Jersey tunnel commissions he advised that, if the New York-New Jersey vehicular tunnel be so ventilated that persons passing through the tunnel would be exposed to not more than 4 parts of carbon monoxide in 10,000 parts of air for not longer than 45 minutes, they would experience no ill effects. Supplementary experiments, carried out by us in connection with our studies at the Bureau of Mines experiment station at Pittsburgh, substantiated this advice.

In continuing our studies on low concentrations of carbon monoxide, we made a few experiments in a specially constructed gas chamber where the conditions could be accurately controlled. The factors investigated were: (1) The effect of long exposure in low concentrations of carbon monoxide; (2) The effect of strenuous exercise; (3) The effect of high temperature and humidity in low concentrations of carbon monoxide.

The "work" was done on a bicycle ergometer and calculated as foot pounds. The carbon monoxide was made by dropping formic acid into hot concentrated sulphuric acid and purified by passing through a soda-lime canister. The composition of the gas used was 99.0 per cent. and 1 per cent. air. Analyses were made of blood samples taken from time to time, and record was kept of the body temperature, pulse, and general condition of the subject.

Results of the tests are summarized below:

With the subject at rest:

1 The exposure for 6 hours to 2 parts of CO in 10,000 caused:

(a) saturation of 16 to 20 per cent. of the hemoglobin of the blood with CO;

(b) very mild subjective symptoms of CO poisoning at the end of the test;

(c) no noticeable effects after the test.

2 The exposure to 3 parts of CO caused:
(a) saturation of 22 to 24 per cent. of the hemoglobin with CO after 4 hours, and 26 to 27 per cent. after 5 hours;
(b) symptoms at the end of 2 hours absent; after 4 hours, mild effects attributed to CO poisoning; and after 5 hours, moderate effects;
(c) after-effects of 4 hours' exposure mild; of 5 hours' exposure, moderate.

3 The exposure to 4 parts of CO in 10,000 caused:
(a) saturation of 15 to 19 per cent. of the hemoglobin with CO at the end of 1 hour, and 21 to 28 per cent. at the end of 2 hours;
(b) after-effects, moderate to marked.

With the subject exercising strenuously:

1 The exposure for 1 hour to 2½ parts of CO in 10,000 caused:
(a) saturation of 14 to 16 per cent. of the hemoglobin with CO;
(b) moderate symptoms of CO poisoning at the end of the test;
(c) after-effects mild to moderate.

2 The exposure for 1 hour to 4 parts of CO in 10,000 caused:
(a) saturation of 23 per cent. of the hemoglobin with CO;
(b) moderate symptoms of CO poisoning;
(c) moderate after-effects.

Further details are given in a recent report issued by the Bureau.

Synthetic Indigo.—A new synthesis of indigo is described by G. C. BAILEY and R. S. POTTER (*Jour. Am. Chem. Soc.*, 1922, xliv, 215–216). They used fumaric acid, prepared from maleic acid, as their raw material. Fumaric acid was converted into symmetrical dibromsuccinic acid by treatment with bromine and glacial acetic acid, under pressure at a temperature of 100° C. for 7 hours. The dibromsuccinic acid was converted into symmetrical dianilidosuccinic acid by treatment with aniline. Three hundred grams of an equimolecular mixture of sodium hydroxide and potassium hydroxide were dehydrated by heating at a temperature of 450° C. for two and one-half hours in a closed agitated iron pot. Then 30 grams of sodamide were added, and a current of anhydrous ammonia was

passed through the pot. Twenty-five grams of sodium dianilidosuccinate were added to the fused mass, and a temperature ranging from 230° to 240° C. was maintained for one and one-half hours. The resulting mass was dissolved in water, and air was blown through the solution. A precipitate of indigo formed and was collected on a filter. The indigo was 96.5 per cent. pure, and possessed satisfactory dyeing properties. The amount of indigo obtained was 60.4 per cent. of the theoretical yield from dianilidosuccinic acid, and 25 per cent. of the theoretical yield from fumaric acid.

J. S. H.

Death Valley, California. (*U. S. Department of Agriculture, Press Service Sheet No. 200.*)—Ten years of records obtained at the United States Weather Bureau substation at Greenland ranch in Death Valley, California, indicate that this is the hottest region in the United States, and probably on earth. The average of extreme maximum temperature reported to the Department of Agriculture since 1911 has been 125° F. At Greenland ranch temperatures of 100° F. or higher occur almost daily during June, July, and August. The hottest month on record is July, 1917, when the mean temperature was 107.2° F. But the temperature of 134° F. observed on July 10, 1913, is believed by meteorologists to be the highest natural-air temperature ever recorded with a standard tested thermometer exposed in the shade under approved conditions.

Death Valley is from two to eight miles wide and about 100 miles long, lying between high mountain ranges. It is the deepest depression in the United States, some estimates placing its lowest point at 337 feet below sea level. Greenland ranch is 178 feet below sea level. White people find the midsummer heat intolerable, and even the Indians go up to the Panamint Range during July and August.

The normal annual precipitation in Death Valley is less than two inches. Successful agriculture can not be maintained on less than 15 or 20 inches of annual precipitation without the aid of irrigation. A group of springs serve as the source of irrigation-water supply for Greenland ranch. The water has a temperature of about 100° F. and is only sufficient to irrigate 70 acres. Four crops of alfalfa are gathered each year. The principal product of the ranch is dressed meat, but experiments are being made in raising poultry and in growing vegetables, dates, citrus, and deciduous fruits.

THE FRANKLIN INSTITUTE.

(Proceedings of the Stated Meeting held Wednesday, April 19, 1922.)

HALL OF THE FRANKLIN INSTITUTE,

PHILADELPHIA, April 19, 1922.

DR. WALTON CLARK, President of the Institute, *in the Chair.*

The Board of Managers submitted its report. The report recorded the election to membership of:

Joseph M. Emerick, Esq., Office Manager—John T. Windrim, Architect,
Commonwealth Building, Philadelphia, Pennsylvania.

John Irvin Bright, Esq., Architect,
Otis Building, Philadelphia, Pennsylvania.

J. Heron Crosman, Jr., Esq., Vice-President,
The Bell Telephone Company of Pennsylvania, Philadelphia, Pennsylvania.

Frederic H. Garrigues, Esq., Mathematician,
Penn Mutual Life Insurance Company, Philadelphia, Pennsylvania.

L. H. Kinnard, Esq., President,
The Bell Telephone Company of Pennsylvania, Philadelphia, Pennsylvania.

Howard S. Levy, Esq.,
Philadelphia, Pennsylvania.

Louis Williams McKeehan, Esq., B.S., M.S., Ph.D., Research Physicist,
Western Electric Company, New York City, New York.

Frederick J. Pearson, Esq., Technical Engineer,
Chicago, Illinois;

lectures before the Sections by

R. B. Moore, D.Sc., Chief Chemist,
Bureau of Mines,
Department of the Interior,
Washington, District of Columbia,

on "Low Temperature Research and the Cryogenic Laboratory of the Bureau of Mines"; by

Harvey Fletcher, Ph.D.,
Research Laboratories,
The American Telephone and Telegraph Company,
New York City, New York,

on "Nature of Speech and Its Perception"; by

E. W. Washburn, Ph.D.,
Department of Ceramics,
University of Illinois,
Urbana, Illinois.

on "Physical Chemistry and Ceramics";

lecture before the Stated Meeting March 15, 1922, by

M. deKay Thompson, Ph.D.,
Associate Professor of Electrochemistry.
Massachusetts Institute of Technology,
Cambridge, Massachusetts,

on "Recent Progress in Applied Electrochemistry";
and additions to the library by gift, 26 volumes and 139 pamphlets, by purchase, 73 volumes and by binding, 28 volumes.

The Chairman then announced that the next business of the meeting would be the presentation of medals and certificates to gentlemen whose inventions had been examined by the Committee on Science and the Arts and found worthy of recognition by the Institute. He then recognized Mr. Henry B. Allen, who introduced Mr. Samuel T. Freas, of Trenton, New Jersey, recently awarded the Edward Longstreth Medal for his "Interlocking" Tooth Saw.

Mr. Allen said, "Mr. President: One often hears it said of a successful invention how surprising it is that no one thought of it before, it being so simple. In reality surprise should be expressed that anything so simple was thought of at all.

"In the present instance there has been produced a detachable tooth saw for cutting metal, the success of which results largely from its remarkable simplicity. This simplicity of design is due to no mere chance, but has behind it years of concentrated application to the study of metal cutting.

"The use of circular saws for cutting metal, while not new, is becoming increasingly extended. The inserted tooth saw allows faster and more economical cutting, but the intense stresses set up in the teeth of a saw doing heavy work has made it a difficult problem to hold the teeth in place, and at the same time readily removable and replaceable. The medalist has done this by a very simple but unusual combination of wedges.

"It is in consideration of this excellence of design resulting in the extended and successful commercial use of this saw that The Franklin Institute, through the Committee on Science and the Arts, has awarded the Edward Longstreth Medal to Samuel T. Freas, Esquire, of Trenton, New Jersey.

"It gives me great pleasure to present to you Mr. Freas, inventor of the 'Interlocking' Tooth Saw."

The President presented the Medal, accompanying Certificate and Report to Mr. Freas, who thanked the Institute for the high honor conferred upon him.

Mr. George H. Benzon, Jr., was then recognized and described the Automatic Die Cutting Machine invented by Mr. Joseph F. Keller, of Brooklyn, New York, who had recently been awarded the Edward Longstreth Medal for his invention.

Mr. Benzon addressed the President as follows: "Mr. President: I believe I can say without fear of contradiction that the keynote of modern manufacturing is quantity production. I may also state that of the articles coming under this classification, a very large percentage are made in metal dies or molds. From the foregoing statements a slight appreciation of the importance of metal dies in the industrial world may be formed.

"Mr. Joseph F. Keller, who is with us this evening, has developed a series of machines known as the Keller Automatic Die Cutting Machines, which

automatically cut the metal dies to their various shapes under the supervision of an average mechanic, where ordinarily the skill of an especially trained man is required. These machines do the work more efficiently and save a considerable amount of time.

"In view of the improvements made by Mr. Joseph F. Keller in the art of mechanical die cutting, the Institute acting through its Committee on Science and the Arts has awarded to him the Edward Longstreth Medal.

"Mr. President, I have the honor of introducing Mr. Joseph F. Keller."

The President presented the Medal, accompanying Certificate and Report to Mr. Keller, who thanked the Institute for the high honor conferred upon him.

The paper of the evening on "The Structure and Building of Atom Nuclei" was presented by Dr. William D. Harkins, of the Department of Chemistry, University of Chicago, Chicago, Illinois. The speaker stated that the atom seems to consist of a central positive nucleus, which plays the part of the sun in the solar system, surrounded by a system of negative planetary electrons. The number of these planetary electrons is equal to the charge of the nucleus.

If an atom loses one or more of its planetary electrons, it easily picks up other negative electrons and becomes restored to its initial condition. However, if the nucleus is broken up, no known experimental means suffice to put it together again; so the atom is said to disintegrate only when the nucleus is broken apart.

The lightest of all atoms, that of hydrogen, consists of one hydrogen nucleus with one positive charge, or what will be termed a positive electron which carries practically all of the mass of the atom together with a planetary system consisting of one negative electron. All of the heavier or complex atoms seem to have nuclei which are built up from a number of positive electrons equal to the atomic weight, together with about half as many negative electrons which serve to bind them together.

The conditions under which an atom nucleus is stable were discussed. The principal stability conditions are very simple. The most stable nuclei contain an even number of both negative and positive electrons, the more important condition being that the number of negative electrons shall be even. Light atoms are in general more stable than heavy ones, though there are more types of heavy than of light atoms.

One of the most important of the stability conditions is that as the positive charge on the nucleus becomes larger the ratio of negative to positive charges in the nucleus must also become larger. That is, as the net positiveness of the nucleus increases the relative negativeness must also increase.

If the complete history of the building of atoms could be unfolded, it would undoubtedly be seen to be one of fundamental importance in determining the present condition of the stars, and also of the surface of the earth and the organisms which dwell upon it.

That elements may be split apart into isotopes has been found recently. The first isotopes separated, those of chlorine, were exhibited. The subject was illustrated by lantern slides, models and specimens.

A unanimous vote of thanks was extended to the speaker and the meeting adjourned.

R. B. OWENS,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, April 5, 1922.)

HALL OF THE INSTITUTE,
PHILADELPHIA, April 5, 1922.

DR. JAMES BARNES *in the Chair.*

The following reports were presented for final action:

No. 2783: "Interlocking" Tooth Saw. The Edward Longstreth Medal to Mr. Samuel T. Freas, of Trenton, New Jersey.

Literature: On the recommendation of the Sub-Committee on Literature, the following awards were made:

No. 2787: The Howard N. Potts Medal to Dr. Richard B. Moore, Chief Chemist, Bureau of Mines, Department of the Interior, Washington, D. C., for his paper on "Helium, Its History, Properties and Commercial Development" printed in the issue of the JOURNAL OF THE FRANKLIN INSTITUTE for February, 1921.

No. 2788: The Certificate of Merit to Rear Admiral W. H. G. Bullard, Director, Naval Communication Service, Navy Department, Washington, D. C., for his paper on "The Application of Radio to Navigation Problems," printed in the JOURNAL OF THE FRANKLIN INSTITUTE for June, 1921.

No. 2789: The Certificate of Merit to Dr. Charles E. Mendenhall, of the University of Wisconsin, Madison, Wisconsin, for his paper on "Aeronautic Instruments," printed in the JOURNAL OF THE FRANKLIN INSTITUTE for January, 1921.

The following reports were presented for first reading:

No. 2780: Rich System of Detecting and Extinguishing Marine Fires.

No. 2786: Postage Meter.

R. B. OWENS,
Secretary.

SECTIONS.

Section of Physics and Chemistry.—A meeting of the Section was held in the Hall of the Institute on Thursday, March 2, 1922, at eight o'clock, P.M., with Dr. Thomas D. Cope in the Chair. The minutes of the previous meeting were approved as read.

Colonel E. Lester Jones, A.M., Director of the United States Coast and Geodetic Survey and Commissioner, International Boundary, United States-Alaska and Canada, delivered a lecture on "Surveying from the Air." The standard surveying methods of the United States Coast and Geodetic Survey were described; and the history of aerial surveying was outlined. An account was given of the aerial surveys of Atlantic City, the coast of New Jersey and the delta of the Mississippi River. The lecture was illustrated with lantern slides. The com-

munication was discussed; on motion of Mr. W. H. Fulweiler a vote of thanks was extended to Colonel Jones; and the meeting adjourned.

JOSEPH S. HEPBURN,
Secretary.

Section of Physics and Chemistry.—A meeting of the Section was held in the Hall of the Institute, Thursday, March 23, 1922, at eight o'clock, P.M., with Dr. Harry F. Keller in the Chair. The minutes of the previous meeting were read and approved.

R. B. Moore, Sc.D., Chief Chemist, Bureau of Mines, United States Department of the Interior, delivered an address on "Low Temperature Research and the Cryogenic Laboratory of the Bureau of Mines." A résumé was given of the history of the liquefaction of gases with special reference to air, hydrogen, and helium. The application of low temperature work to physical constants and to the knowledge of the constitution of matter was described. The lecture was illustrated by lantern slides and experiments.

The communication was discussed at length; a vote of thanks was extended to Doctor Moore; and the meeting adjourned.

JOSEPH S. HEPBURN,
Secretary.

Electrical Section.—A meeting of the Section was held on Thursday evening, March 30, 1922, at eight o'clock, with Dr. James Barnes in the Chair.

Harvey Fletcher, Ph.D., of the Research Laboratory of the American Telephone and Telegraph Company and Western Electric Company, Incorporated, New York City, presented the paper of the evening on the "Nature of Speech and Its Perception." The relative importance of various frequency ranges in spoken sounds were translated into the electrical form. The electrical communication circuit used was the most nearly perfect ever devised. Electric wave filters were employed to eliminate at will various parts of the scale of speech frequencies. The intelligibility, characteristic of any condition of speech communication, was determined by test employing scientifically chosen syllables. The experimental results given exhibited in quantitative terms the effects of using limited ranges of frequency to transmit speech. The data was analyzed so that the specific action upon each vowel and consonant was plainly evident. The evidence thus obtained would seem to demand a modification of the present theories of speech.

The subject was illustrated by numerous experiments and lantern slides.

A unanimous vote of thanks was extended to the speaker and the meeting adjourned.

ALFRED RIGLING,
Acting Secretary.

ASTON LECTURES.

Under the Auspices of the Bartol Research Foundation of The Franklin Institute.

Dr. F. W. Aston, F.R.S., Fellow of Trinity College, Cambridge, England, delivered a series of lectures on "Atomic Weights and Isotopes" in the lecture hall of the Institute on the afternoons of March 6th to 10th, inclusive.

The first lecture was delivered on the afternoon of Monday, March 6th, at 3.30 o'clock. Mr. Coleman Sellers, Jr., Vice-president of the Institute, welcomed Doctor Aston, and Dr. Arthur W. Goodspeed, Professor of Physics, University of Pennsylvania, M.F.I., presided. The subject of this lecture was "The Atomic Nature of Matter" and the following is a brief synopsis:

The atomic structure of matter generally; ancient speculation. "Is matter infinitely divisible?" Views of Newton and the early chemists. Dalton's atomic theory. Size and number of atoms illustrated by repeated subdivision of a cube of lead. Relative delicacy of methods for the detection of elements. Modern methods of detection of single atoms.

Dr. Augustus Trowbridge, Professor of Physics, Princeton University, M.F.I., presided at the second lecture on Tuesday, March 7th. Subject, "Discharge of Electricity in Gases." Special attention was given the following:

Nature of low-pressure discharge. Crookes' dark space, its length at different pressures in various gases, variation with current intensity. Primary dark space. Distribution of potential in the Crookes' dark space. Failure of theories suggested. Cathode rays and positive rays.

Mr. W. H. Fulweiler, Chief Chemist, United Gas Improvement Company, Philadelphia, M.F.I., presided at the third lecture on Wednesday, March 8th. Subject, "Positive Rays and Their Analysis." Special attention was given to the following:

Sir J. J. Thomson's parabola method of analysis of positive rays. First proof of the approximate identity of weights of atoms of the same element. Abnormal chemical compounds. Triatomic hydrogen. Neon and its parabolas. Attempts to separate its components. Failure of fractional distillation, success of diffusion experiments. Analogy to Soddy's radioactive isotopes.

"The Mass-spectrograph" was the subject of the lecture on Thursday afternoon. The following topics were considered:

Problem of increasing the accuracy of positive ray analysis. Methods of focussing the rays. Principle of the mass-spectrograph. Details of its construction and experimental technic. Standard lines. Lines of the first, second and higher orders. Mass-spectra of the chemical elements neon, chlorine, etc. Remarkable results with krypton, xenon and mercury. Mass-spectra of the alkali metals. Isotopes and the periodic law.

Dr. Joseph S. Ames, Director of the Physical Laboratory, Johns Hopkins University, Baltimore, M.F.I., presided.

The closing lecture of the series on "Isotopes and the Structure of the Atom" was given on Friday afternoon. The following topics were considered:

The whole number rule. Abnormality of hydrogen. Method of measuring masses of hydrogen and helium by "bracketing." The electrical structure of atoms, protons and electrons. The Rutherford atom and its later development

by Bohr and Langmuir. The spectra of isotopes. Their chemical identity and its probable explanation. Separation of isotopes. Possible methods. Work of Bronsted and Hevesy. Suggested explanation of the constancy of the composition of chemical elements and conjectures as to their evolution.

Dr. Arthur L. Day, Geophysical Laboratory, Washington, D. C., M.F.I., presided.

Physicists and Chemists were in attendance from all parts of the Eastern, Southern and Midwestern States and Canada.

A paper by Doctor Aston, giving a summary of these lectures, appears in this number of the Journal.

MEMBERSHIP NOTES.

ELECTION TO MEMBERSHIP.

(*Stated Meeting, Board of Managers, April 12, 1922.*)

LIFE MEMBERSHIP.

MR. JOSEPH MCCLARY EMERICK, 1501 Commonwealth Building, Philadelphia, Pennsylvania.

RESIDENT MEMBERSHIP.

MR. JOHN IRWIN BRIGHT, Otis Building, Philadelphia, Pennsylvania.
MR. J. HERON CROSMAN, JR., The Bell Telephone Company of Pennsylvania, Philadelphia, Pennsylvania.
MR. FREDERIC H. GARRIGUES, 6806 N. 11th Street, Philadelphia, Pennsylvania.
MR. L. H. KINNARD, The Bell Telephone Company of Pennsylvania, Philadelphia, Pennsylvania.
MR. HOWARD S. LEVY, 1424 N. 15th Street, Philadelphia, Pennsylvania.

NON-RESIDENT MEMBERSHIP.

DR. L. W. MCKEEHAN, 463 West Street, New York City, New York.
MR. FREDERICK J. PEARSON, 25 E. Washington Street, Chicago, Illinois.

CHANGES OF ADDRESS.

MR. E. G. BASHORE, care of DR. J. D. BASHORE, Shippensburg, Pennsylvania.
MR. A. W. K. BILLINGS, Canadian and General Finance Company, Limited, Toronto, Canada.
MR. J. V. BOSTWICK, St. Davids, Pennsylvania.
DR. FRANK K. CAMERON, 46 Bransford Apartments, Salt Lake City, Utah.
MR. FRED DENIG, 87 Magnolia Avenue, Jersey City, New Jersey.
MR. KERN DODGE, Morris Building, Philadelphia, Pennsylvania.
MR. BYRON E. ELDRED, 30 Nassau Road, Great Neck, Long Island, New York.
MR. C. E. FOSTER, 120 St. George's Road, Ardmore, Pennsylvania.
MR. GEO. W. MCCLELLAND, 3112 Troost Avenue, Kansas City, Missouri.
MR. FREDERICK C. FEARING, 2112 Delancey Place, Philadelphia, Pennsylvania.
MR. LEON W. NICHOLS, Glenolden, Pennsylvania.

MR. JOSEPH J. RHOADS, Miller Park, Franklin, Pennsylvania.
MR. F. A. SEIBERLING, 41 East Mill Street, Akron, Ohio.
MR. C. F. SULZNER, 236 S. W. 2nd Street, Miami, Florida.

NECROLOGY.

William Colwell Carnell was born in Philadelphia on June 11, 1869, and died on March 12, 1922. After the usual training in the public schools he entered the Central High School in February, 1885, from which he was graduated in February, 1889, with the degree of Bachelor of Arts. In the following year he became a student at Lehigh University and was one of the graduates of the class of 1894 in Analytical Chemistry. Immediately after graduation he entered the chemical laboratories of the Hill Steel Plant at Chicago, where he remained three years. The next four years he spent as chemist for the Nelson-Morris Company at the Chicago Stock Yards. From 1901 to 1914 he was connected with Charles Lennig and Company and the three following years he spent with the Harrison Paint Company. In 1917 he was engaged with the Rohm and Haas Company and in the following year he again became connected with the Charles Lennig Company. He was Vice-president of the latter company at the time of his death.

For several years he was instructor in Chemistry in the evening classes of Temple University.

He was a member of numerous clubs, the American Institute of Chemical Engineers, the American Chemical Society and the Society of Chemical Industry. His interest in the Institute began when a minor and continued throughout his entire life. He became a member of The Franklin Institute on July 11, 1913, and was elected to membership in the Committee on Science and the Arts in January, 1916.

Henry Hess was born at Darmstadt, Germany, in 1862, and died in Atlantic City, New Jersey, on March 24, 1922.

He was educated in private schools in New York City and then returned to Germany for the purpose of entering one of its high schools. During his career as a designer and mechanical engineer he was connected with the Pond Machine Tool Company, Watervliet Arsenal, Niles Tool Works, and the Bureau of Construction and Repair of the United States Navy. He was consulting engineer and managing director of the German Niles Tool Works at Berlin and founder and president of the Hess-Bright Manufacturing Company, Hess-Steel Company, and Hess-Ives Company.

He was a member of the American Society of Mechanical Engineers, the Society of Automobile Engineers, the American Institute of Mining Engineers, American and International Societies for Testing Materials, the Institute of Automobile Engineers, the Engineers' Club of Philadelphia, the Society of German Engineers and a number of clubs. He became a member of The Franklin Institute on October 8, 1904.

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BOOK NOTICES.

METRIC SYSTEM FOR ENGINEERS, by Charles B. Clapham, Hons. B. Sc. Eng. 178 pages and index, small 8vo. New York, E.P. Dutton and Company, 1922. Price $6.00 net.

The author tells us that the object of this book is an explanation of the metric system from the technical point of view, but he does not enter positively into the question of the compulsory adoption of it by any nation. His views are, of course, entirely directed toward the agitation on the subject in Great Britain. It is stated that the metric system is now regularly taught in British schools, but it is still regarded by many as an alien topic, and he says, further, that even those who use the units in the drafting room or workshop are rarely confident in calculating with them. This language sounds a little strange to American readers, who are so familiar with the decimal units of our currency, but it appears to be true that the curious arithmetical relations of pounds, shillings and pence have developed in the English a lack of facility with decimal calculations in some ways. Herbert Spencer, who, as is well known, was a bitter opponent of the metric system and even pursues it after death, for he left an endowment to pay for literature

opposing it, remarks in one of his attacks that a certain British accountant had told him that he had just finished a very elaborate review of a monetary transaction in English currency, and that he was very glad it was not in francs. Yet it is a curious fact, that may be mentioned in this connection, that Spencer looks with some favor upon the compulsory adoption of a duodecimal system which he declares would be much more convenient than the decimal one. He is right on this point. A base of twice 6 would make a more convenient system of calculation than the present base of twice 5. Discussions of the latter type, however, may be regarded as purely academic. It is an interesting fact that of all the great nations of the world, the English speaking ones are alone in direct antagonism to the metric system. It is not the decimal character alone that is the advantage, for after all, as regards measures length and surface, we can meet the problem by dividing the inch into tenths, hundredths and thousandths, which is as convenient as dividing the centimetre into similar fractions. A great value that the introduction of the metric system has, and the one which has made it especially popular among chemists and physicists, is its simplification of the data in weights and volumes. The simple relation of the gram to the litre, of the centimetre to the litre and the ease with which questions of percentage of standard solutions of different strengths may be calculated, has rendered this system an indispensable method in the laboratory and for many years all leading British and American chemists have entirely relied upon it. Apart from the simplification of these relations the fact that by means of the system we can get rid of the confusion of the old units is no small gain. An ounce, for example, may be a troy ounce, an avoirdupois ounce or a fluid ounce. When an English chemist speaks of a gallon, he means the imperial gallon of ten avoirdupois pounds. When an American chemist speaks of a gallon he means the United States or wine gallon (if permissible to allude to wine nowadays) of 8.33 pounds and the serious confusions that may thus arise are easily seen.

Turning to the work itself under consideration, it is an elaborate collection of data covering the whole subject of the conversion to and fro between the metric system and the older, or so-called, British system. Fourteen pages are devoted to a brief survey of the controversy which, as well known, has been extremely bitter in both Great Britain and America. The complexity of the older system is presented and it is rather interesting to note in the brief account of the history of the metric system that a decimal basis for a system of weights and measures was first proposed by James Watt in 1783, and that the decimal basis of United States currency, which has been of such immense economy to the American people, is due to Thomas Jefferson. It is, of course, known to all who are interested in this field that the original intention of the metric system was to obtain a natural absolute basis, the one-forty millionth of the earth's meridian. It is now known that the arc of the earth's surface which was measured for this purpose, was not correctly determined, and consequently the standard metre is an arbitrary quantity. A list of industries in England in which the output is wholly metric is given, and is certainly encouraging. It is impossible here to review to any extent the chapter on the controversy, but the general opinion of the author seems to be that most of the objections are not as serious as believed by those who

urged them. Taken all in all, the book contains a vast amount of carefully selected and arranged details, which will undoubtedly aid very materially those who desire to employ the system and are not familiar with the general principles, or do not care to acquaint themselves with these principles, but wish to be able to pass quickly from one unit in a given case to its equivalent in another.

The book is well printed, the many tables are clear and there are numerous illustrations. Taken as a whole it is an argument in favor of the general adoption of the metric system.

HENRY LEFFMANN.

GAS CHEMISTS HANDBOOK, compiled by the Chemical Committee of the Technical Section of the American Gas Association. Second edition, 586 pages and index, 113 illustrations, numerous tables, 8vo. New York, The American Gas Association. Price, $6.00 net.

The production of illuminating gas is important not only for the product itself, which affords a very convenient method of lighting and heating, especially the latter, but also for the by-products which have been of late years, as is well known, the source of a great variety of important substances. The present work is devoted to the analytical methods required of the gas engineer and not to the technology or business problems. These latter, of course, are very important features of the industry, but the chemist is not directly concerned with either of them. The book is an extensive presentation of the methods of analysis of all the materials used and produced in the manufacture of gas. It begins with a chapter on raw materials including coal, coke, and oil and the substances used for the purification of the products.

The analytical procedures in regard to the products, both the gas itself and the other materials, are then considered in a chapter of over 200 pages. The examination of tar from both water- and coal-gas receive full attention. Two of the by-products that are now increasing in importance in consequence of the probable early exhaustion of the great supply of combined nitrogen from Chili, namely, ammonia and cyanogen, are especially studied in the impurities in the gas. Some which are of the most serious nature, and not always easy to remove, are then taken up, including especially hydrogen sulphide and other sulphur compounds, which latter include carbon disulphide.

An interesting impurity in the gas and one that would have seemed impossible in the early days of gas engineering is iron carbonyl. This volatile compound of iron is formed in the high temperature of the retort and is a serious impurity under certain conditions. It appears that the process for its determination, which is here given, is strongly recommended, although not in general use. It essentially depends upon the oxidation of the product by nitric acid or bromine. The compound is present in minute quantities, as a rule, which renders it necessary to treat rather large volumes of gas in order to obtain satisfactory determinations.

The complexity of the gas industry is well illustrated by the extension and variety of the last chapter which discusses the accessory materials used in the construction and operation of the works. Among these water receives very careful consideration. The methods employed are those endorsed by the American Public Health Association and hence will require no particular

criticism. It is to be noted, however, that these methods include the test of hardness by the use of soap solution. It would seem that in any well-equipped laboratory the calcium and magnesium of a given sample could be very conveniently determined by the ordinary process of analysis and thus the somewhat uncertain method of soap solution be left to one side.

The other substances that are treated are paint and paint materials, lubricants, solder, bearing metals, brass and bronze, pipe deposits, refractory materials, cement and lime and ferrous metals, under which last term is included the analysis of steels, pig and cast iron. The methods used for these purposes are those of the American Society for Testing Materials. Numerous tables are given for conversions of various kinds and for analytical calculations. A fairly good index closes the volume.

The book represents an up-to-date collection of analytical methods involved in the operation of a gas plant. Its compilers are men who have worked in these lines for years and are thoroughly familiar with the requirements of the industry and it is a valuable addition to the literature of chemical engineering. It is a well-printed and well-illustrated volume.

HENRY LEFFMANN.

THE FOREST PRODUCTS LABORATORY. A Decennial Record. Published by the Decennial Committee, Edward F. Weiss, Chairman. 8vo, 196 pages and many illustrations. Madison, Wisconsin. Price, $1.75.

This laboratory for studying the characters and utilization of forest products was established at the University of Wisconsin in 1910, and it has been deemed appropriate to celebrate the decennial anniversary by the publication of a volume summarizing the work done. The operation of the laboratory is carried out by members of the Staff of the Forest Service of the Department of Agriculture. The book is a handsomely printed and well-illustrated volume, and is an opportune contribution to one of the most important fields of national industry.

Undoubtedly, the preservations of our forests and the utilization as far as possible of all the products, is a matter worthy of most careful and energetic research. The text of the work is introduced by a chapter showing the relation of wood to civilization. The contrast between countries that maintain appreciable amounts of forest and those that allow unrestricted devastation, is alone sufficient to show the value of forest conservation. Such conservation does not in any way limit the reasonable use of the forest. The work of the forest laboratory is very extensive, one of the most important being the study of preservatives. Much of the volume is, as might be expected, taken up with personal matters, but illustrations of the various appliances of the laboratory are inserted and show the wide range of work that can be carried on. It is a "forest products" laboratory, not a centre of forestry management, so that questions of conservation of forests as such, fire prevention, systematized culling and other matters that are involved in the national and state forestry services are not presented. The work will be especially valuable in showing how wood that was formerly considered worthless can be applied to useful purposes.

HENRY LEFFMANN.

ZIRCONIUM AND ITS COMPOUNDS. By Francis P. Venable. 148 pages, bibliography and index, 8vo. The Chemical Catalog Company, New York. Price, $2.50 net.

This is another volume of the A.C.S. monograph series. Devoted entirely to the chemistry of one element, and that element commonly regarded as among the rarer and less important substances, the volume shows the high development of specialism in the science, and the manner in which research is bringing into prominence elements hitherto deemed worthy of merely a line or two in our standard text-books. At present, zirconium can scarcely be considered a rare element, its compounds being found in appreciable amount in many parts of the world, yet it is in far less supply than such substances as aluminum and silicon. It was identified as a distinct element by Klaproth in 1789. Separation of the metal in the pure condition is difficult on account of its high affinity for oxygen and the presence of other somewhat analogous elements in its ores. It is classified in the fourth group, which begins with carbon, hence the typical oxide is ZrO_2 and the chloride $ZrCl_4$. It, however, shows some analogies to uranium on the one hand and to bismuth and antimony on the other. Thus it tends to form salts in which oxygen partly takes the place of the ordinary acid radicles, which is noted in connection with uranium, and many of its salts are promptly decomposed by water with the formation of oxy- or hydroxy salts as occurs with bismuth and antimony. One would be inclined to expect such anomalies to occur rather with elements of high atomic weights, but the atomic weight of zirconium is about 91, the highest figure being that found by Venable and Bell in 1917, 91.76. This figure has not been accepted by the International Committee, which gives 90.6.

Many uses of zirconium compounds are mentioned in the book, but it seems that in each field the use is rather limited. The high melting point of the oxide, and its permanence in the air led to its being recommended as a substitute for lime in the oxyhydrogen light. Robert Hare, in 1820, first observed the brilliancy of the light emitted by it. In 1868, du Motay, who gave a good deal of attention to the improvement of street-lighting in Paris, used it in lighting the Tuileries. Welsbach made early use of zirconia in his mantles, but the thoria-ceria mixture has taken its place. Considerable application has been made of the high melting point of zirconia in the manufacture of refractories. Alloys of zirconium with iron seem to have some valuable properties.

Professor Venable has collected in this book an immense amount of information, set forth in clear, vivid language, and has practically combed the literature clean for the data. He has not overlooked the comparatively recently noted interesting fact that the hypophosphite is sensitive to light. An extended bibliography, arranged chronologically, is appended. It must be noted that, while the other publications of this series use the spelling "sulfide," "sulfate," etc., the present work has gone back to older spelling. Perhaps this has been an oversight; perhaps the author has insisted on the forms, but in any event it is to be regretted that a retrograde step has been taken.

The book maintains the standard already reached by this excellent and valuable series.

HENRY LEFFMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS. Technical Note No. 93. The Background of Detonation. By Stanwood W. Sparrow, Automotive Power Plants Section, Bureau of Standards. 17 pages, quarto. Washington, Committee, April, 1922.

The tendency of a fuel to detonate often renders it unsuitable for use in high compression engines. Detonation appears to be closely related to the explosion pressure and this depends upon the temperature and pressure of the charge before combustion. This paper discusses the influence upon explosion pressures and hence upon detonation of changes in compression ratio, spark advance, compression pressure and degree of scavenging. Consideration is given to the loss in power entailed by various methods of eliminating detonation. Preignition is shown to be another obstacle to the use of high compression ratios and its influence is compared with that of detonation.

Report No. 134, Performance of Maybach 300-Horsepower Airplane Engine. By S. W. Sparrow. 11 pages, illustrations, quarto. Washington, Government Printing Office, 1922.

This report deals with the results of a test made upon a Maybach engine in the altitude chamber of the Bureau of Standards, where controlled conditions of temperature and pressure can be made to simulate those of the desired altitude.

The results of this test lead to the following conclusions: From the standpoint of thermal efficiency the full-load performance of the engine is excellent at densities corresponding to altitudes up to and including 15,000 feet. The brake mean effective pressure is rather low, even at wide-open throttle. This tends to give a high weight per horsepower, inasmuch as the weight of many engine parts is governed by the size rather than the power of the engine. At part load the thermal efficiency of the engine is low. Judged on a basis of performance the engine's chief claim to interest would appear to lie in the carburettor design, which is largely responsible for the excellent full-load efficiency and for its poor part load efficiency.

Report No. 135, Performance or B. M. W. 185-Horsepower Airplane Engine. By S. W. Sparrow. 10 pages, illustrations, quarto. Washington, Government Printing Office, 1922.

This pamphlet presents the results of a test made upon a B. M. W. engine in the altitude chamber of the Bureau of Standards.

A remarkably low value of fuel consumption—0.41 pound per b.h.p. hour—is obtained at 1200 r.p.m. at an air density of 0.064 pounds per cubic foot and a brake thermal efficiency of 33 per cent. and an indicated efficiency of 37 per cent. at the above speed and density. In spite of the fact that the carburettor adjustment does not permit the air-fuel ratio of maximum economy to be obtained at air densities lower than 0.064, the economy is superior to most engines tested thus far, even at a density (0.03) corresponding to an altitude of 25,000 feet.

The brake mean effective pressure even at full throttle is rather low. Since the weight of much of the engine is governed more by its piston displacement than by the power developed, a decreased mean effective pressure usually necessitates increased weight per horsepower. The altitude performance

of this engine is, in general, excellent, and its low fuel consumption is the outstanding feature of merit.

Report No. 138, The Drag of C Class Airship Hull with Varying Length of Cylindrical Midships. By A. F. Zahm, R. H. Smith and G. C. Hill. 10 pages, illustrations, plates, quarto. Washington, Government Printing Office, 1921.

A model of the C class airship hull, when severed at its major section and provided with a cylindric mid-body of variable length, had its air resistance increased about in proportion to the length of the mid-body up to three diameters, and in about the manner to be expected from the increase of skin friction on this variable length. For greater length the drag increased less and less rapidly.

As usual for such models, the drag for any fixed length, at 20 to 60 miles an hour, is accurately of the parabolic form RaV^n, and hence the drag coefficient is of the hyperbolic form CaV^{n-2}, where n is slightly less than 2.

The variation of C with length is stated in the conclusion.

Report No. 139, Influence of Model Surface and Air-flow Texture on Resistance of Aerodynamic Bodies. By A. F. Zahm. 6 pages, quarto. Washington, Government Printing Office, 1922.

In this pamphlet is given an analysis of the resistance equation

$$D = \rho L^2 V^2 f\left(\frac{VL}{v}\right).$$

It is shown that the expression $f\left(\frac{VL}{v}\right)$ applies only to a special case of uniform air flow and model surface texture. In order to obtain comparable results under various conditions it is necessary to use a more general form of the resistance equation, such as

$$D = \rho L^2 V^2 f\left(\frac{VL}{v}, \frac{l}{L}, \frac{v}{V}, \frac{V}{a}, \frac{V^2}{Lg}\right)$$

in which $\frac{l}{L}$ is a measure of the model surface texture, and $\frac{v}{V}$ a measure of the air-flow texture. These two functions have particular application in the comparison of tests from different aerodynamical laboratories.

PUBLICATIONS RECEIVED.

Organic Chemistry or Chemistry of the Carbon Compounds, by Victor von Richter; edited by Prof. R. Anschütz and Dr. H. Meerwein. Volume II, chemistry of the carbocyclic compounds. Translated from the eleventh German edition by E. E. Fournier d'Albe. 760 pages, 8vo. Philadelphia, P. Blakiston's Son and Company, 1922.

Le Principe de Relativité et la Théorié de la Gravitation. Leçons professées en 1921 et 1922 a l'École Polytechnique et au Muséum d'Histoire Naturelle par M. Jean Becquerel. 342 pages, illustrations, 8vo. Paris, Gauthier-Villars et Cie., 1922. Price 25 francs.

Problèmes et Exercises d'Électricité Générale, par P. Janet. 253 pages, illustrations, 8vo. Paris, Gauthier-Villars et Cie., 1921.

Gas Chemists Handbook. Compiled by the Chemical Committee, Technical Section of the American Gas Association, March 1, 1922. Second edition, 608 pages, illustrations, plates, 8vo. New York, American Gas Association, no date. Price $6.00

Applied Calculus. An introductory text-book by F. F. P. Bisacre. 446 pages, illustrations, portraits, 12mo. New York, D. Van Nostrand Company, 1921. Price $3.75

Metric System for Engineers, by Charles B. Clapham. 181 pages, illustrations, 8vo. The Directly Useful Technical Series. New York, E. P. Dutton and Company, 1922. Price, $6.00.

Essai d'Optique sur la Gradation de la Lumière, par Pierre Bouguer. 131 pages, illustrations, 16mo. Paris, Gauthier-Villars et Cie., 1921.

The Forest Products Laboratory. A decennial record, 1910-1920. 196 pages, illustrations, portraits, quarto. Madison, Wisconsin, published by the Decennial Committee, 1921. Price $1.75

The Conservation of Natural Gas in Kentucky, by Willard Rouse Jillson, Director and State Geologist of the Kentucky Geological Survey. First edition. 152 pages, illustrations, maps, 12mo. Louisville, Kentucky, John P. Morton Company, 1922.

The Coal Industry in Kentucky. An historical sketch by Willard Rouse Jillson, Director and State Geologist of the Kentucky Geological Survey. 87 pages, illustrations, maps, 12mo. Frankfort, The State Journal Company, 1922.

National Advisory Committee for Aeronautics: Technical Notes, No. 25, Center of Pressure Coefficients for Aerofoils at High Speeds, by W. S. Diehl. 2 pages, plates, quarto. No. 87, Hydrostatic Test of an Airship Model by the Aeronautics Staff, Construction Department, Navy Yard. 15 pages, plates, photographs, quarto. No. 89, The Choice of the Speed of an Airship by Max M. Munk. 8 pages, plate, quarto. Technical Paper, No. 91, Notes on Propeller Design. The Energy Losses of the Propeller-I, by Max M. Munk. 10 pages, quarto. No. 92, Full Scale Determination of the Lift and Drag of a Seaplane by Max M. Munk. 5 pages, plate, quarto. Washington, Committee, 1922.

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PHILADELPHIA